Empirical research on the model of elementary school mathematics teaching materials' difficulty

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Difficulty is the important indicator for evaluating mathematics teaching materials, which has referential significance for analyzing the students' burden of school work, writing and revising mathematics teaching materials and implementing mathematics curriculum reform. However, very little attention of research at home and abroad had been paid to it. The purpose of this study was to build a new model of the difficulty for characterizing the elementary school mathematics teaching materials scientifically. Therefore, a questionnaire was designed, based on the literature analysis, expert interviews and related research conclusions, to survey the key factors affecting elementary school mathematics teaching materials' difficulty. In data analysis, some statistical methods were employed, such as exploratory and confirmatory factor analysis. A new model of elementary school mathematics teaching materials' difficulty was constructed with nine key affecting factors and three difficulty dimensions including the difficulty of content, worked examples and exercises.
Keywords: elementary school mathematics, teaching materials' difficulty, model

1. Introduction

Chinese students' outstanding achievements in a variety of international mathematics competition or test coexisting with their heavy school work burden have caused many scholars' concern about the differences between eastern and western mathematics education. One view of the reason for Chinese students' outstanding achievements and heavy school work burden is the mathematics learned by Chinese students is more difficult than other countries'. Mathematics teaching materials are the most direct media math teachers and students contacting the mathematics curriculum content. Determining mathematics teaching materials' difficulty is, therefore, the most direct way to judge the difficulty of the math learned by students.

The purpose of this study is to build a model to characterize scientifically elementary school mathematics teaching materials' difficulty. To this end, a questionnaire was designed, based on the literature analysis, expert interviews and related research conclusions, to survey the factors affecting elementary school mathematics teaching materials difficulty. Then the key factors, dimensions and structure of elementary school mathematics teaching materials' difficulty were selected or determined through the statistical analysis methods such as factor analysis.

2. Method

（1）Selection of respondents

In this study, a sample of 1454 math educators from 5 provinces in China completed a questionnaire. As illustrated in Table 1, this sample is representatives for all kinds of math educators from various region of China, including 1244 elementary school mathematics teachers (ESMT), 70 elementary school mathematics teaching and research staffs (ESMTRS), 23 authors of elementary school mathematics textbooks

(ESMTA), and 117 mathematics educational experts (MEE) from universities. Authors of elementary school mathematics textbooks are selected from authors of the West Normal Version of elementary school mathematics textbooks. Mathematics educational experts are mainly from the researchers of Mathematics education in Chongqing and Sichuan universities.

Table 1 Distribution of respondents

	ESMT	ESMTRS	ESMTA	MEE	Total
Chongqing	240	17	16	47	320
Sichuan	262	15	5	33	315
Henan	119	14	2	8	143
Jiangxi	160	11	0	13	184
Shandong	463	13	0	16	492
Total	1244	70	23	117	1454

(2) Research instrument

In order to fully explore the difficulty of elementary school mathematics textbooks influencing factors, the study further analyzed the literature of the evaluation and analysis of teaching materials, and then conducted semi-structured interviews. Based on the literature analysis, expert interviews and related research conclusions, 16 influencing factors of elementary school mathematics textbooks difficulty were initially identified in this study, including content quantity(CQ), content demand(CD), content materials selection (CMS), content presentation manner(CP)， content correlation(CC), content sequence(CS), worked example quantity(WQ), worked example demand(WD), worked example complexity(WC), type of worked example (WT)and its fit in with content(WFC), and exercise quantity(EQ), exercise demand(ED), exercise complexity(EC), type of exercise(ET) and exercise matching with the worked example(EMW). The “questionnaire on the influencing factors of elementary school mathematics textbooks difficulty” was, then, designed. The questionnaire was scored with five-point Likert scale ranging from “very unimportant(1)” to “very important（5）”. The questionnaire’s Cronbach reliability coefficient α is 0.908, which indicating that the questionnaire has good internal consistency.1454 questionnaires were distributed and collected, and the rate of collection was 100%. 116 questionnaires were excluded because of not answering, serious lack of data, and incomplete basic information (mainly from questionnaires answered by elementary school mathematics teacher). 1338 questionnaires were valid, and the rate of validation was 92%. Data were mainly input and analyzed with Excel, SPSS19.0 and Amos 17.0.

3. Results and discussion

In view of the purpose of this study, recognition degree analysis, exploratory factor analysis and confirmatory factor analysis were used on the data of questionnaires. Recognition degree was defined as the average of all surveyed data of one factor. In other words, a factor’s recognition degree = the sum of all surveyed data of this factor/the total number of valid questionnaires. So the values of recognition degree range from 1 to 5.

(1)Recognition degree of each influencing factor of elementary school mathematics textbooks difficulty

Each of the recognition degree of 16 influencing factors of elementary school mathematics textbooks difficulty is equal to or greater than 3.5, as shown in Figure 1. Content quantity (CQ) has the highest recognition degree, reaching 4.2, and exercise quantity (EQ) has the lowest recognition degree 3.5, followed by the recognition degree of worked example quantity (WQ) 3.6.

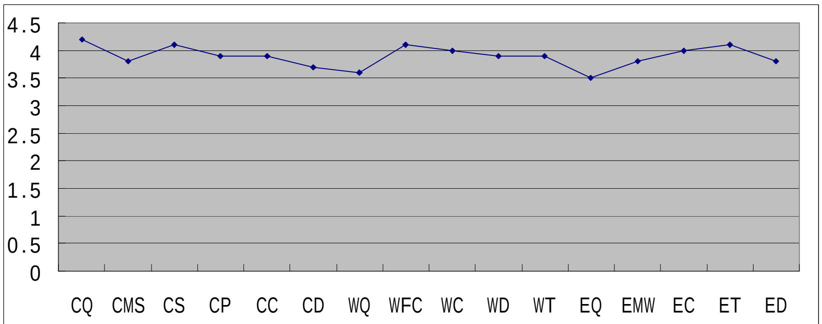

Fig 1.Recognition degree of each influencing factor

(2)Group difference of each influencing factor's recognition degree

Overall no significant differences of each influencing factor's recognition exist among different groups of mathematics educators as shown in Figure 2. Content materials selection (CMS) and worked example complexity (WC) have the biggest difference between authors of elementary school mathematics textbooks (ESMTA) and elementary school mathematics teaching and research staffs (ESMTRS). The difference value of type of worked example (WT) and exercise quantity (EQ) is 0.6. The difference value of other factors is not more than 0.5, wherein the smallest difference is 0.2 of the content quantity (CQ), followed by 0.3 of the content sequence (CS).

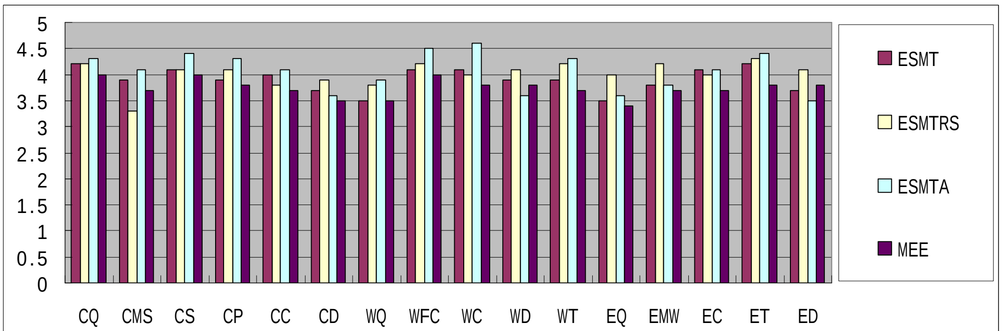

Fig 2.Group difference of each influencing factor's recognition degree

(3)Factor analysis

In view of the needs of the exploration and confirmation, data of each group of mathematics educator were randomly divided into two halves, one for exploratory factor analysis, and the other for confirmatory factor analysis.

I. Exploratory factor analysis

At the 0.05 level of significance, the 16 influencing factors of elementary school mathematics textbooks difficulty are correlated. The correlation of exercise quantity

(EQ) 0.022 and content correlation (CC) 0.021 don't arrive at the 0.01 level of significance, and then the exercise quantity (EQ) is deleted due to its lowest recognition degree in above analysis.In the KMO and Bartlett test for remaining 15 influencing factors, the KMO value is 0.880, which means components exist in them. Worked example quantity (WQ) is removed in the coming factor analysis because of its minimum commonality 0.401 and lower recognition degree in the previous analysis.

The other 14 influencing factors are analyzed with methods similar to above analysis, and then analyzed with principal components method and varimax method of rotation. The components are extracted according to whether their eigenvalue greater than 1 or not. The rotated components matrix is shown in Table 2. Content sequence (CS) is located between the influencing factors of exercise and worked example, and its loadings in factor 1 and factor 2 are very close. Although its recognition degree is high with small group difference in previous analysis, content sequence (CS) is still removed. We can also see from Table 2, the loadings of exercise complexity (EC) in factor 1 and factor 2 are very close, and then exercise complexity (EC) is removed in further factor analysis similar to content sequence (CS).

Similar to the previous analysis, considering the commonality of various factors, loadings differences in extracted factors, recognition degree and its group differences, content materials selection (CMS), type of worked example (WT) and content correlation (CC) are also deleted. That is to say, 7 influencing factors are deleted, 9 influencing factors are remained in the further factor analysis.

Table 2 Rotated component matrix [a]

	component		
	1	2	3
ED	.857	.038	.161
ET	.754	.204	.151
EMW	.645	.340	.204
EC	.593	.572	.152
CS	.585	.513	.215
WC	.271	.772	.061
WFC	.021	.727	.246
WD	.486	.612	.213
WT	.437	.609	.214
CD	.349	.033	.728
CP	.110	.059	.697
CQ	.270	.456	.642
CMS	-.031	.401	.598
CC	.361	.409	.468

Table 3 Rotated component matrix [a]

	成分		
	1	2	3
ED	.882	.053	.175
ET	.796	.209	.090
EMW	.643	.461	.172
WC	.198	.829	.098
WFC	.051	.786	.153
WD	.442	.633	.245
CP	-.006	.151	.826
CD	.298	.111	.773
CQ	.349	.447	.512

Result of exploratory factor analysis showed that three extracted factors (as shown in Table 3) basically reflect the influencing factors of exercises, worked examples, and content difficulty. The loadings of every influencing factor in extracted factors are relative concentrated. The explained total variance is 68.559%. The commonality of each influencing factor except content quantity (CQ) is higher than 0.6. In the KMO and Bartlett test, the KMO value is 0.826 which indicating that there are common factors in 9 influencing factors.

II. Confirmatory factor analysis

In order to verify whether this structural factors model fit with the actual data, and to explore the relationship of the three dimensions of elementary school mathematics textbooks' difficulty, the other half data of the questionnaire are processed with confirmatory factor analysis.

It is found in the first-order confirmatory analysis that the correlations among the three dimensions of elementary school mathematics textbooks' difficulty are higher, and the structural model fits the sample data better. Namely, there is a higher-order latent variable can explain these three dimensions. So second-order confirmatory factor analysis is used to make sure of this latent variable and its relationship with three dimensions in previous statement.

The structural equation graph of elementary school mathematics textbooks' with standardized estimates is shown in Figure 3, and the main indicators of model fit are summarized in Table 4. Each error variance is positive, and the indicators are up to standard, which means the model and fit the actual data better. It can be found in Figure 3 that the loading of elementary school mathematics textbooks' difficulty (T'D) in worked examples' difficult (W'D) is 0.90, followed by exercises' difficulty (E'D) 0.87, and the minimum loading 0.77 of contents' difficulty (C'D). Worked examples' difficult (W'D) are impacted greatly by worked example demand (WD), the influencing of worked example complexity(WC) and its fit in with content(WFC) are basically the same and relatively the less. The loadings of Exercises' difficulty (E'D) in three influencing factors are generally equal. Contents' difficulty (C'D) is mainly affected by content quantity (CQ).

Tab 4 main indicators of model fit

Indicator	CMIN/DF	RMR	GFI	RMSEA
Value	1.194	0.036	0.977	0.030

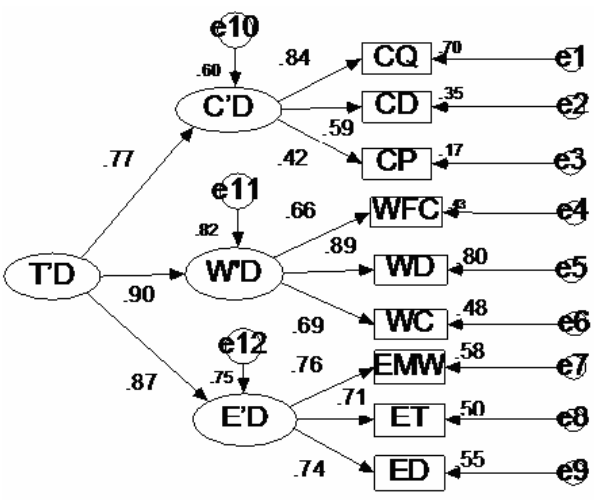

Fig 3. Structural equation graph

4. Conclusion

1. Key influencing factors and its relationships of elementary school mathematics textbooks' difficulty

It can be known from previous factor analysis that elementary school mathematics textbooks' difficulty has 9 key influencing factors which can be roughly summarized to three dimensions: the contents' difficulty (C'D), worked examples' difficulty (W'D) and exercises' difficulty (E'D). The influencing factors in each dimension mainly relate to demands on students, selection of content and its material, means of instructional processing, and their relationship.

We can learn from the result of confirmatory factor analysis, worked examples' difficult (W'D) have the maximum loading of elementary school mathematics textbooks' difficulty, followed by exercises' difficulty (E'D), and then is contents' difficulty, the ratio of them are approximately 4:3:3. Those results are consistent with the real situation of elementary school mathematics textbooks which are mainly made up by worked examples and exercises, and contents often appear after a specific worked example.

The ratio of worked example demand (WD), worked example complexity (WC) and its fit in with content (WFC) in worked examples' difficult (W'D) are approximately 4:3:3. This shows worked examples as the main means of content instructional processing, its difficulty are mainly subject to its demands on students. That is to say, worked examples with high complex and contents' difficulty may be not difficult if its demands on students are lower.

The contributions of three influencing factors to exercises' difficulty are basically the same, about one-third each. Exercises as the main way of students' participation, its difficulty are equally affected by its demands on students, students' prior mathematical reality, and their match with worked examples.

The ratio of content presentation manner (CP), content demand (CD) and content quantity (CQ) in contents' difficult (C'D) are approximately 2:3:5. This is basically in line with the features of elementary school mathematics content, such as foundation and little topics, and with the high recognition degree of content quantity (CQ). That is to say, contents' difficulty will significantly increase if the content of elementary school mathematics is added.

2. Model of elementary school mathematics textbooks' difficulty

Referring to the existing related research on the quantitative description of the difficulty of mathematics curriculum and math problems, the model of elementary school mathematics textbooks' difficulty can be expressed mathematically as follows:

$$T'D = 0.77C'D + 0.90W'D + 0.87E'D$$

$$C'D = 0.84CQ + 0.59CD + 0.42CP; W'D = 0.89WD + 0.69WFC + 0.69WC; E'D = 0.76EMW + 0.71ET + 0.74ED$$

The standardized model can be approximately expressed as follows:

$$T'D = 0.30C'D + 0.36W'D + 0.34E'D$$

$$C'D = 0.45CQ + 0.32CD + 0.23CP; W'D = 0.40WD + 0.30WFC + 0.30WC; E'D = 0.34EMW + 0.32ET + 0.34ED$$