Challenges of Robust Statistical Characterization Using Digital Image Correlation Technique for Structural Applications

Mark R. Gurvich
United Technologies Research Center, East Hartford, CT, USA
E-mail: GurvicMR@utrc.utc.com

Abstract

Digital Image Correlation (DIC) technique is a relatively new experimental optical method to measure the entire deformation field at structural surfaces as a function of applied load. Generated distributions are based on individual measurements for a significant population of randomly distributed speckles and corresponding statistical approximations. Practical engineering interest, however, is rather focused on strains defined as derivatives of displacements. Considering inevitable scatter of individual measurements, the statistical part of this problem is reduced, therefore, into characterization of quasi-random derivatives (strains) for measured quasi-random functions (displacements). The work lists major challenges associated with robust implementation of this statistical problem. A set of simple benchmarking statements is proposed for evaluation of computational efficiency for corresponding statistical analysis. Since exact solutions for these statements are already known, they can be used for accuracy assessment of current and new DIC-related statistical implementations.

Key words: DIC, non-uniformity, random functions, reliability assessment

1. Problem Statement

Digital Image Correlation (DIC) technique is an advanced experimental method used for characterization of displacement fields in structures or materials under different types of loads including mechanical and thermal conditions, impact, forced or natural vibrations, etc. In contrast with other more traditional methods of experimental mechanics, DIC is based on optical measurements of movement for a very large population of points (speckles) applied to the surface of considered structures. Schematic representation of the DIC idea is shown in Figure 1, where individual deformation of each speckle can be potentially captured and quantified according to selected coordinate system. Then the entire population of the displacements can be statistically post-processed to generate quasi-analytical approximations representing the entire field of displacements $U_x(x,y)$ and $U_y(x,y)$.

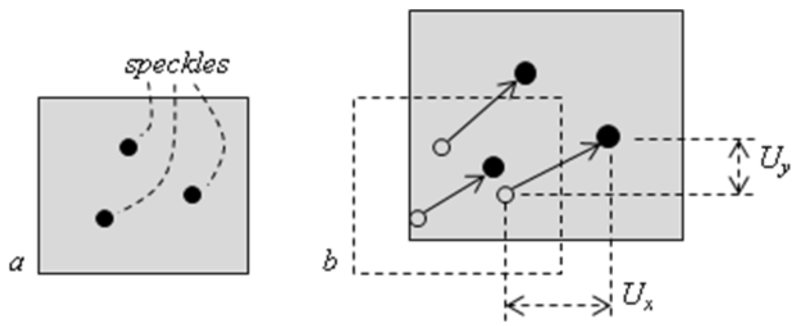

Figure 1. Scheme of DIC characterization showing a) undeformed and b) deformed states.

Practical engineering interest, however, is primarily focused on local changes in deformation to make conclusions regarding strength and structural integrity. These local geometrical changes are quantified as strains and defined as derivatives of the displacements. For example, strains ε_{xx} in direction x are defined as

$$\varepsilon_{xx}(x,y) = \partial U_x(x,y)/\partial x \qquad (1)$$

These strain fields can also be used to calculate distributions of stresses using known characteristics of stiffness and used for comparison with material strength allowables.

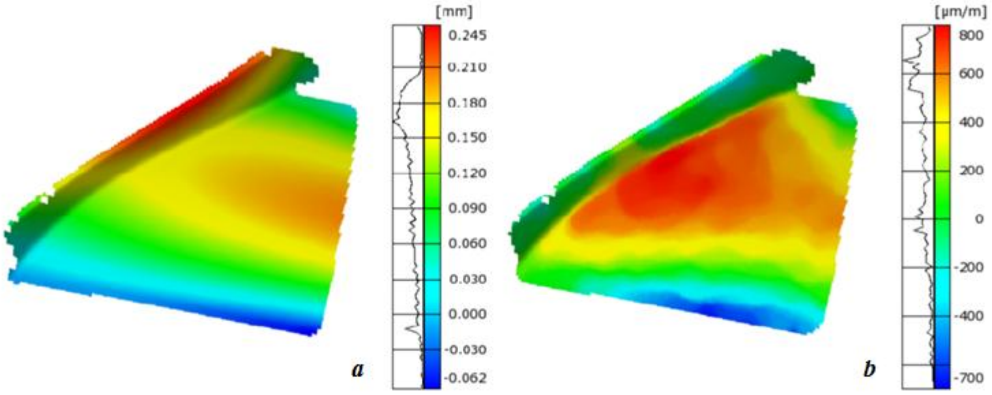

Figure 2. Representative example #1 of a) measured field of displacements U_x and b) calculated field of strains ε_{xx}.

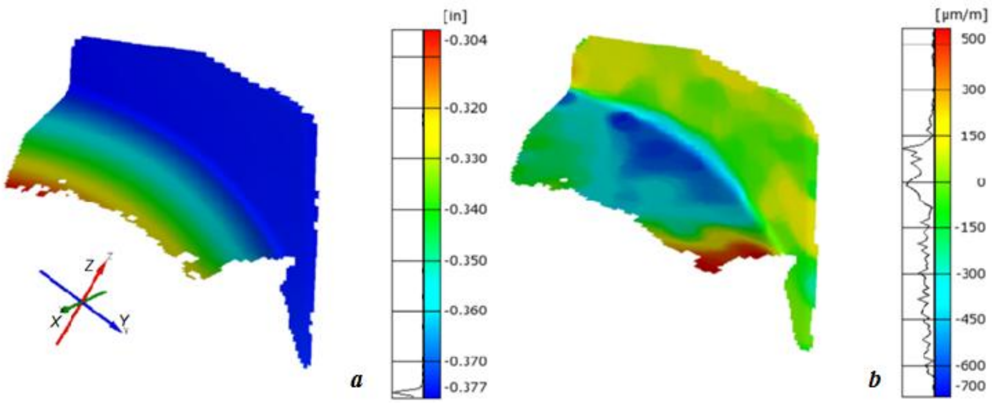

Figure 3. Representative example #2 of a) measured field of displacements U_x and b) calculated field of strains ε_{xx}.

Typical engineering implementations of DIC are based on consideration of large populations with thousands of randomly distributed speckles. Quantification of their movement can be associated with measurement noise and inevitable variability requiring careful statistical post-processing. From the viewpoint of DIC-based post-processing, characterization of directly measured fields of displacements is probably the easiest part. In this case, practically any reasonable approximation techniques can be successfully applied. Two examples of measured and post-processed fields of *displacements* are shown in Figures 2a and 3a for two segments of a representative structure. Very smooth distributions are demonstrated in spite of quite significant statistical variability and 3D geometrical complexity of the structure.

Corresponding *strain* distributions, however, indicate certain sensitivity to statistical variation and methods of post-processing treatment (Figures 2b and 3b). This type of observations is not unusual in DIC analysis, where accuracy and robustness of strain characterization may require development of more efficient statistical methods, especially, in cases of relatively high randomness of measurement. In other words, the challenge of robust statistical assessment of *derivatives* for a measured *random function* is the focus of the present work. More specifically, the objective is to i) define *the statistical* problem of DIC-based strain characterization capturing inevitable randomness of measurements; and ii) propose metrics to quantify accuracy and robustness of corresponding statistical solutions.

2. Approach

The following three major questions should be addressed by DIC-related statistical solutions: if a random population of points representing dependence $y(x)$ is known,

- are additional *physical assumptions* needed to calculate its derivative $y'(x)$?
- is there a *unique* solution for the derivative?
- what is the level of *confidence* for generated predictions of the derivatives?

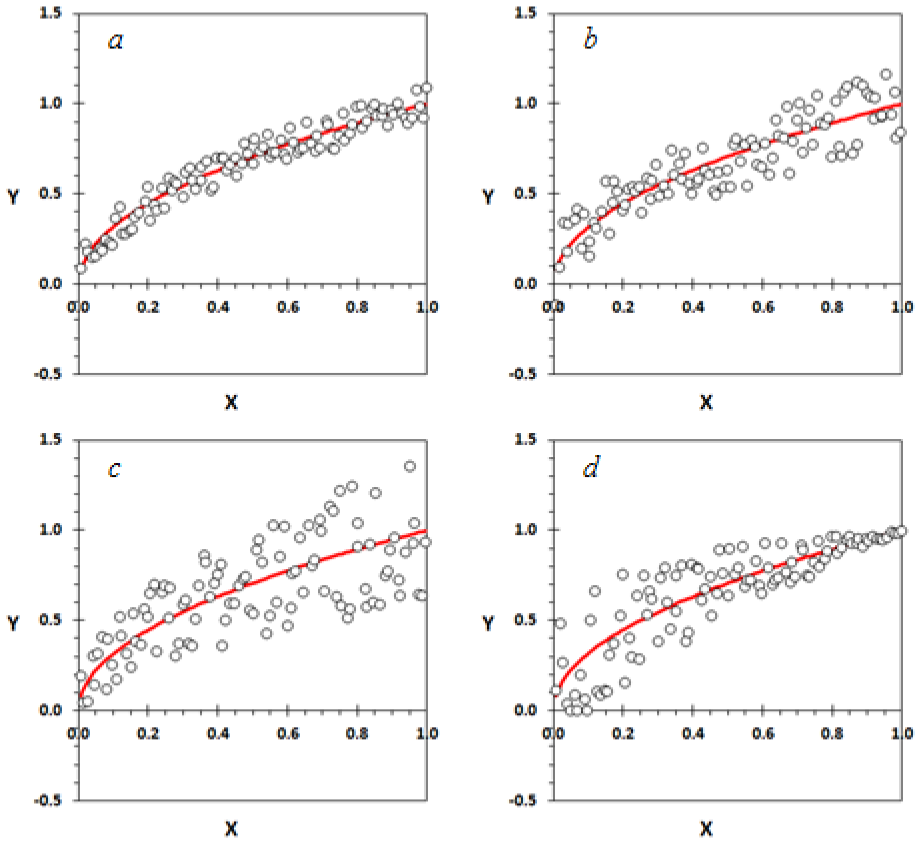

Figure 4. Examples of the problem statements with different definitions of variability: a) uniform small; b) uniform large; c) non-uniform with constant coefficient of variation; d) non-uniform with linear dependence.

This work does not propose statistical solutions for DIC-related challenges, but rather defines an approach of quantitative evaluation of potential solutions. The proposed approach is based on assumption that the exact analytical function is already known and serves as a reference metric for accuracy assessment. Then random implementations of the analytical function are introduced using Monte-Carlo simulation (MCS) for a known pattern of expected variability. An example shown in

Figure 4a illustrates this approach in a graphical form, where the red line represents an introduced analytical function, and the points represent a MCS-generated population of corresponding quasi-random implementations. These random implementations are used further as input for statistical analysis according to the applied solution. Calculated output in form of predicted approximations of the *derivative* is finally compared with the originally introduced analytical equations to make conclusions regarding accuracy and efficiency of the analysis.

3. Benchmarking Problems

Proposed benchmarking problems are aimed to mimic different major types of static, quasi-static and dynamic deformation. According to practical engineering interest, four major scenarios are suggested in this work:

a) monotonic function with uniform variability (Figures 4a,b);
b) monotonic function with non-uniform variability (Figures 4c,d);
c) non-monotonic function with singularity and uniform variability (Figures 5a,b);
d) non-monotonic function with discontinuity and uniform variability (Figures 5c,d).

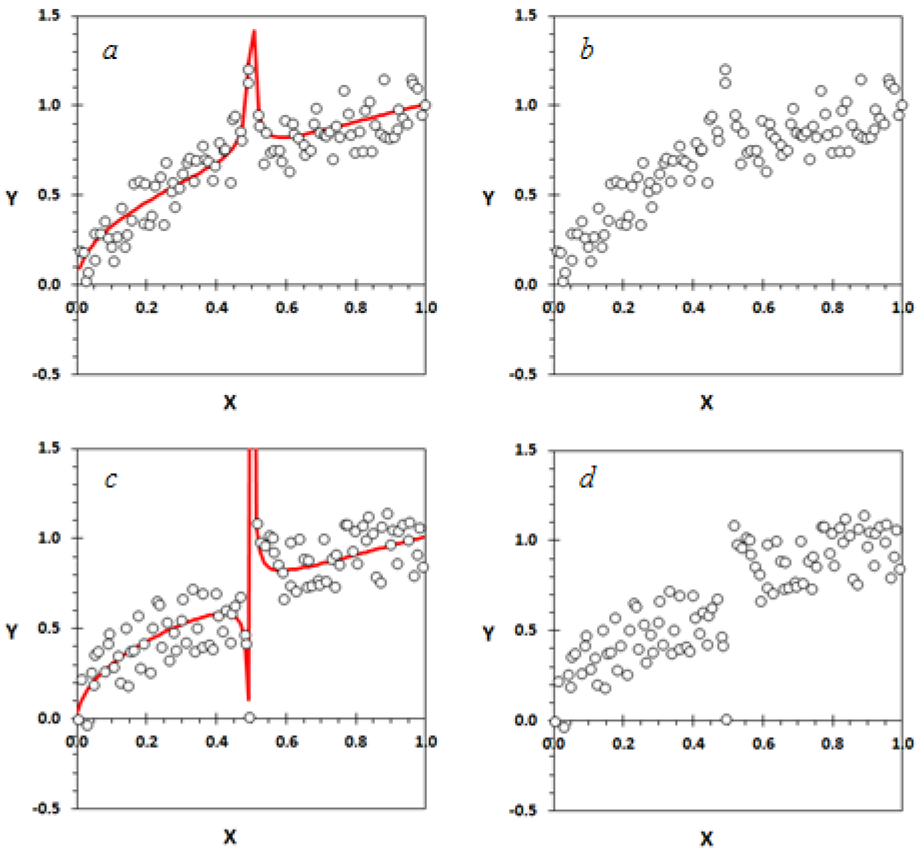

Figure 5. Examples of the problem statements with different local gradients (clarifications in the text).

Scenarios (a,b) mimic deformation of homogeneous bodies, and cases (c,d) represent deformation in the vicinity of internal damages due to expected strain concentrations in the vicinity of crack tips. For all scenarios, reference random function is defined as

$$\tilde{y}(x) = y(x) + \Delta\tilde{y}(x) \tag{2}$$

where $y(x)$ indicates the mean function (the red curve); and $\Delta\tilde{y}(x)$ indicates the variability with zero mean (sign "~" above the variable shows its random character). (For simplicity, they are defined as 1-D statements since corresponding multi-dimensional variants can be easily generalized if needed). Analytical descriptions of the mean functions for considered cases are given below:

scenario (a,b): $$y(x) = Ax^p; \quad y'(x) = Apx^{p-1}; \tag{3}$$

scenario (c): $$y(x) = Ax^p + \frac{B}{|x-C|}; \tag{4}$$

$$\text{if } x < C,\ y'(x) = Apx^{p-1} + \frac{B}{(x-C)^2}; \text{ if } x > C,\ y'(x) = Apx^{p-1} - \frac{B}{(x-C)^2}; \tag{5}$$

scenario (d): $$y(x) = Ax^p + \frac{B}{(x-C)}; \quad y'(x) = Apx^{p-1} - \frac{B}{(x-C)^2}; \tag{6}$$

Statistical variability of function $\Delta\tilde{y}(x)$ can be defined by either uniform or normal probabilistic distributions. In case of scenarios (a,c,d), the variability is independent of x. In case of scenario (b), two benchmarking implementation can be considered. The first one is variability with constant coefficient of variation (Figure 4c). The second representative implementation of non-uniform variability (Figure 4d) can be described, for example, by a linear dependence.

4. Representative Results

Numerical implementations of considered scenarios are illustrated in Figures 4-5 at A = 1; p = 0.5; B = 0.005; C = 0.5; and uniform variability of $\Delta\tilde{y}(x)$ (in all examples, 100 uncorrelated points per population were generated using MCS). It is easy to see that higher level of variability (e.g., Figure 4b vs. Figure 4a) can significantly complicate definition of derivatives. Similarly, non-uniformity of the variability (Figure 4c,d) can provide additional statistical challenges for robust or even unique assessment of derivatives $y'(x)$. The biggest challenges, however, are associated with areas of high gradients (Figure 5). Indeed, some irregularities can be observed in zones close to singularities ($x \approx 0.5$) as shown in Figures 5b,d. However, accurate estimation of derivatives in these zones can be potentially too sensitive to methods of statistical treatment. Unfortunately, zones with significant local gradients represent major interest in experimental mechanics as indications of internal damages.

5. Conclusions

Statistical challenges of DIC-based experimental characterization are defined. It is shown that major obstacles are associated with robust statistical characterization of derivatives (strains) for populations representing discrete measurements of displacements, especially, in case of high variability. Benchmarking problems with known exact solutions are proposed as metrics of accuracy for corresponding statistical methods. These generalized problems represent scenarios similar to practical cases of experimental mechanics. Future efforts can be focused on quantitative assessment of existing statistical methods on examples of the benchmarking problems.

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