The Generalized Filter Trading Rule under Time Varying Volatility

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Abstracts

Filter trading rule is a technical trading strategy with a long history dated back to the pioneer work by Alexander (1961). The filter trading rule generates a sequence of buy/sell trading signals according to the following principle. If the asset price moves up at least $100\delta\%$ from a low, the signal sequence will start with a buy signal. The rule then suggests to buy and hold the asset until the price moves down at least $100\delta\%$ from a subsequent high, at which time a sell signal is generated and the rule will suggest to sell and go short. It is interesting to observe that there is a mathematical equivalence between filter rule and the CUSUM technical control charts. CUSUM chart which is dated back even earlier to Page (1954) was proved to be Wald's sequential probability ratio test (SPRT) and the assumption for its derivation is that the observations are independent. However, in financial markets, various kinds of dependencies of the asset's returns have been long discovered and widely adopted. This paper is aiming to generalize the ordinary filter trading rule to cater for conditional heteroskedasticity.

Keywords: Filter trading rule, CUSUM, Heteroskedasticity

1 Introduction

Consider a financial asset whose closing price at day t is p_t $(t = 0, 1, 2 \dots)$. Let $y_t = \log(p_t)$ be the log price of the asset and let $x_t = y_t - y_{t-1}$ be the continuously compounding return when the asset is held from the end of day $t-1$ to the end of day t. For a long time, there has been a great interest in devising a mechanical trading rule which dictates when to hold the asset and when to sell short the asset so as to achieve profitable trading. According to Acar and Satchell (2002), a mechanical trading rule consists of a time series of buy/sell signals B_t, $t = 0, 1, 2, \dots$ that can be zero, $+1$ or -1. When $B_t = +1(-1)$, an investor will automatically take a long (short) position from end of day t to end of day $t + 1$. The most well known trading rule practiced by many practitioners in the market is the "moving average trading rule". Its key statistic to determine the trading signal is $p_t - (p_t + p_{t-1} + p_{t-2} + \cdots + p_{t-m+1})/m$ which is a linear function of the prices $p_1, p_2, ..., p_t$. Unlike the moving average trading rule, filter trading rule is a non-linear trading rule in which the buy-sell signals are driven by a non-linear statistic which measures the draw-up or draw-down of the asset prices. Interestingly, this statistic has a long origin that can be dated back to early literature in quality control. As early as 1954, Page (1954) developed a sequential inspection scheme called CUSUM to detect changes in the level of a process. Lam and Yam (1997) pointed out that the CUSUM techniques can be applied to the financial market resulting in the filter trading rule. In light of the financial implications of the

CUSUM statistic, we introduce the notation of DD_t (draw-down) and DU_t (draw-up) to represent these CUSUM as follow:

$$DU_t = \log(p_t) - \min_{0\le\tau\le t} \log(p_\tau) \tag{1}$$

$$DD_t = \max_{0\le\tau\le t} \log(p_\tau) - \log(p_t) \tag{2}$$

Let δ denote the filter size of a filter trading rule. If the initial position at time $t = 0$ is long, the filter trading rule will generate a sell signal once $DD_t > \delta$. On the other hand, if the initial position at time 0 is short, the filter trading rule will generate a long signal once $DU_t > \delta$.

Suppose the return process x_t has mean μ_t and variance σ_t^2. Assume that σ_t^2 is a constant and μ_t may be time varying. Suppose a technical trader takes a long position at the end of day 0 because of his belief that μ_t in the following days will have a positive mean. Despite this belief, he is alert to detect when the mean return will become negative. In other words, he has to carry out a sequential test of the null and alternative hypotheses as $H_0 : \mu_t = \theta_0$ and $H_1 : \mu_t = \theta_1$where θ_0 and θ_1 are two specified constants with $\theta_0 > 0$ and $\theta_1 < 0$. When Wald's SPRT likelihood ratio test is applied to the sequential test above, it can be shown that the resulting likelihood ratio test statistic is the draw-down statistic DD_t. On the other hand, the likelihood ratio test statistic for sequentially testing the other pair of hypotheses as $H_0 : \mu_t = \theta_1$ and $H_1 : \mu_t = \theta_0$ is the draw-up statistic DU_t. This justifies the claim that amongst all technical trading rules, the filter trading rule makes the most statistical sense. For derivation details, one can refer to Yam (1996). However, the derivation of traditional CUSUM test was based on the assumption that series volatility is a constant. In a financial market, various kinds of dependencies of the asset's returns have been long discovered and widely adopted. One of the most important discoveries among them is the conditional heteroskedasticity (CH) which allows the second moment of the return series to be correlated. One natural question that arises is whether the likelihood ratio test statistics resulting from a CH model will be different from those used in traditional CUSUM test or filter trading. This paper is aiming to generalize the ordinary filter trading rules to cater for conditional heteroskedasticity.

2 Filter Trading Rule under CH models

One of the greatest breakthrough in financial time series modeling is the proposal of Autoregressive Conditional Heteroskedasticity (ARCH) models by Engle (1982). It assumes that the conditional variance of the time series is time varying and depends on the squares of the past innovations. Later on, Bollerslev (1986) proposed a generalized ARCH (GARCH) model to let the conditional variance not only depend on the past squared innovations but also depend on the past conditional variances. The conditional heteroskedasticity models have the following general form:

$$\begin{cases} x_t = & \mu_t + \epsilon_t \\ \epsilon_t = & \sqrt{h_t} e_t \\ h_t = & g(\epsilon_{t-1}^2, h_{t-1}, \epsilon_{t-2}^2, h_{t-2}, \ldots) \\ e_t \overset{\text{i.i.d.}}{\sim} & \mathrm{N}(0,1) \end{cases} \tag{3}$$

where μ_t is the conditional mean process that can either be a constant or has very complicated structure; e_t are normal white noises (or innovations) with zero mean and unit variance; h_t is the conditional variance that depends on past squared innovations and/or conditional variances through a known function g. Let us now consider x_t as the log price of a asset and an investor is holding a short position of the asset at time zero. He/she will be interested to decide when the return will start to have a positive mean at which time he /she should switch to a long position. In other words, he/she is interested in testing the following two hypotheses sequentially:

$$\begin{cases} H_0^S: & \mu_t = \theta_1 \text{ for } t = 1, 2, \ldots, n \\ H_1^S: & \mu_t = \theta_1 \text{ for } t = 1, 2, \ldots, v-1 \text{ and } \mu_t = \theta_0 \text{ for } t = v, \ldots, n \text{ for some } v \leq n \end{cases}$$

where $\theta_0 > 0$ and $\theta_1 < 0$ are two specified constants and n is a large positive integer. A sequential test of these two hypotheses starting from $t = 1$will be considered. If H_0^S cannot be rejected at time t, we add a new observation at time $t+1$ and repeat the test. At any time before n, if H_0^S is rejected, then the test will be stopped and a buy signal will be generated. Once the buy signal is generated, the investor will switch to a long position. The buying time is then set to be time zero and he/she will be interested in testing the following hypotheses sequentially:

$$\begin{cases} H_0^L: & \mu_t = \theta_0 \text{ for } t = 1, 2, \ldots, n \\ H_1^L: & \mu_t = \theta_0 \text{ for } t = 1, 2, \ldots, v-1 \text{ and } \mu_t = \theta_1 \text{ for } t = v, \ldots, n \text{ for some } v \leq n \end{cases}$$

where $\theta_0 > 0$ and $\theta_1 < 0$ are two specified constants. For the sake of simplicity, we focus on the sequential test for H_0^S versus H_1^S. Assuming the daily asset return x_i has the GARCH model in Equation 3, the likelihoods under H_0^S are

$$L_{0,t} = \prod_{i=1}^{t} f(x_i|\theta_1, h_i)$$

where $f(x|\theta, h)$ is the density function at x with mean θ and variance h and t will take value sequentially from $1, 2, \ldots, n$. On the other hand, the hypothesis H_1^S is a composite hypotheses with an extra parameter v. The likelihood function under a particular v value is given by

$$L_{1,t,v} = \prod_{i=1}^{v-1} f(x_i|\theta_1, h_i) \prod_{i=v}^{t} f(x_i|\theta_0, h_i)$$

where t will take value sequentially from $1, 2, \ldots, n$. Then the likelihood ratio statistics $\gamma_{S,t}$ for testing H_0^S versus H_1^S is

$$\begin{aligned} \gamma_{S,t} &= \frac{\underset{1 \leq v \leq t}{max} L_{1,t,v}}{L_{0,t}} \\ &= \frac{\underset{1 \leq v \leq t}{max} \left\{ \prod_{i=1}^{v-1} f(x_i|\theta_1, h_i) \prod_{i=v}^{t} f(x_i|\theta_0, h_i) \right\}}{f(x_1|\theta_1, h_1) \cdots f(x_t|\theta_1, h_t)} \end{aligned}$$

Let a be the critical value for the sequential test. In other words, when t runs sequentially from $1, 2, \ldots, n$, the null hypothesis H_0^S will be rejected once $\gamma_{S,t} \geq a$

and the test will be stopped. When $\gamma_{S,t} < a$, an additional observation will be added at $t+1$ and the test continues. Assume that the innovations are normally distributed, we have

$$\begin{aligned}
\log(\gamma_{S,t}) &= \log \frac{\max\limits_{1\le v\le t}\left\{\prod_{i=1}^{v-1}\left(\frac{1}{\sqrt{2\pi h_i}}\exp\left(\frac{(x_i-\theta_1)^2}{2h_i}\right)\right)\prod_{i=v}^{t}\left(\frac{1}{\sqrt{2\pi h_i}}\exp\left(\frac{(x_i-\theta_0)^2}{2h_i}\right)\right)\right\}}{\prod_{i=1}^{t}\left(\frac{1}{\sqrt{2\pi h_i}}\exp\left(\frac{(x_i-\theta_1)^2}{2h_i}\right)\right)} \\
&= \max_{1\le v\le t}\left\{\sum_{i=1}^{t}\left(\frac{(x_i-\theta_1)^2}{2h_i}-\frac{(x_i-\theta_0)^2}{2h_i}\right)-\sum_{i=1}^{v-1}\left(\frac{(x_i-\theta_1)^2}{2h_i}-\frac{(x_i-\theta_0)^2}{2h_i}\right)\right\} \\
&= \sum_{i=1}^{t}\frac{(2x_i-\theta_1-\theta_0)(\theta_0-\theta_1)}{2h_i}-\min_{1\le v\le t}\sum_{i=0}^{v-1}\frac{(2x_i-\theta_1-\theta_0)(\theta_0-\theta_1)}{2h_i} \qquad (4)
\end{aligned}$$

If we further assume that $\theta_0 = -\theta_1 = \theta > 0$, then Equation 4 becomes

$$\log(\gamma_{S,t}) = 2\theta\left\{\sum_{i=1}^{t}\frac{x_i}{h_i}-\min_{1\le v\le t}\sum_{i=1}^{v-1}\frac{x_i}{h_i}\right\}$$

Under a constant variance model where $h_i = \sigma^2$ for all i, the above statistic reduces to

$$\begin{aligned}
\log(\gamma_{S,t}) &= \frac{2\theta}{\sigma^2}\left\{\sum_{i=1}^{t}x_i-\min_{1\le v\le t}\sum_{i=1}^{v-1}x_i\right\} \\
&= \frac{2\theta}{\sigma^2}DU_t. \qquad (5)
\end{aligned}$$

where DU_t is the draw-up statistics of the ordinary filter trading rule. However, under a CH model, we let σ^2 be its long term variance. Then the statistic becomes

$$\begin{aligned}
\log(\gamma_{S,t}) &= \frac{2\theta}{\sigma^2}\left\{\sum_{i=1}^{t}\frac{x_i\sigma^2}{h_i}-\min_{1\le v\le t}\sum_{i=1}^{v-1}\frac{x_i\sigma^2}{h_i}\right\} \\
&= \frac{2\theta}{\sigma^2}DU_t^* \qquad (6)
\end{aligned}$$

where

$$DU_t^* = \sum_{i=1}^{t}\frac{x_i\sigma^2}{h_i}-\min_{1\le v\le t}\sum_{i=1}^{v-1}\frac{x_i\sigma^2}{h_i} \qquad (7)$$

is the test statistics of the generalized filter trading rule. Notice that the statistic DU_t^* is constructed in such a way that DU_t^* and DU_t have the same proportionality constant to the log likelihood ratio. After the alignment, the difference between DU_t and DU_t^* is that returns x_t carry equal weights in DU_t whereas x_t are weighted by the reciprocal of market variances for the computation of DU_t^*. In other words, in the generalized filter trading rule, a fixed given return will be given less weight in days when the market is more volatile. Similarly, when we are testing for a sell signal, the test statistics in generalized filter trading rule becomes

$$DD_t^* = \max_{1\le v\le t}\sum_{i=1}^{v-1}\frac{x_i\sigma^2}{h_i}-\sum_{i=1}^{t}\frac{x_i\sigma^2}{h_i} \qquad (8)$$

3 Empirical Study

In this Section, we will use the market data to compare the performance of the traditional filter trading rule and generalized filter trading rule. The Hang Seng Index Futures contracts (HSIF) are chosen as the underlying asset for trading. One advantage of trading a futures contract is that the transaction cost involved is relatively low. To keep track of the trading profit of the ordinary trading rule is straight forward because there is no need for parameter estimation. However, to keep track of the trading profit of the generalized filter trading rule, we need to estimate the CH parameters. Specifically, we divide the data of HSIF from May 1986 to May 2010 into six equal periods with each period consist of 1000 trading days. The first period is used for parameter estimation only. We then use the estimated parameters to carry out the generalized filter trading in the subsequent period. We then re-estimate the parameters based on the data of the second period and use them for trading in the third period, etc.

Figure 1 shows the comparison of filter trading for conventional filter trading and generalized filter trading through the whole period from May 1990 to May 2010. Figure 2 has the performance comparison for each 4-year short period. The series "SIMPLE" denote the conventional filter trading rule and "GARCH" for generalized filter trading rule. To do comparison, we match the conventional and generalized filter trading results by fixed number of trading signals in each period. That means, chosen fixed number of trades n as 10, 20, 30, or so on, we search for the lowest filter sizes for both ordinary and generalized filter trading that can obtain this number of trades and then compare their trading profits for each n. So the x-axis in the figures are the number of trades n, and y-axis are the total log return of the filter trading for selected periods. We can observe that in this 20 years period, the generalized filter trading performs better than the conventional filter trading for most of the case except for very small number of trading signals. It means the generalized filter trading performs better in capturing asset price's short term tendency and conventional filter trading performs better in capturing asset price's long term tendency. To verify this conclusion, we further divide this 20 year period into five 4-year periods and repeat the comparison for each small period. The results show in Figure 2. However, not every small period support the above conclusion from the whole period. Only the graphs for the periods from May 1990 to May 1994 and from May 1994 to May 1998 is consistent with that of the whole period. While for other three periods, the performance of conventional filter trading rule and generalized filter trading are quite close. One of the reason might be that the market has become more and more efficient these days and making profit is becoming difficult for all technical

Figure 1: Profit for Simple and Generalized Filter Trading from 1990 to 2010

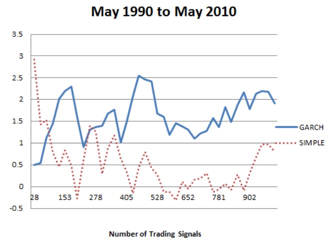
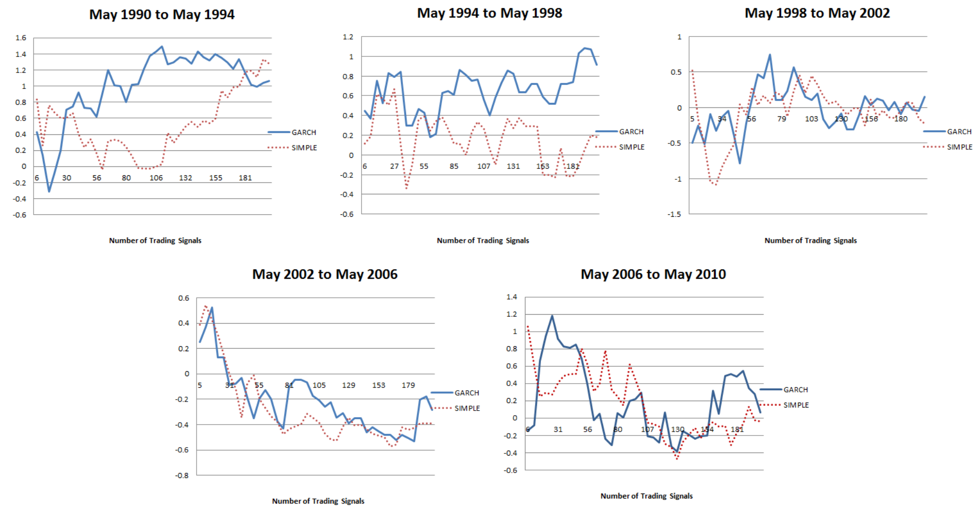


Figure 2: Profit for Simple and Generalized Filter Trading from Each 4-year Period



trading rules. The other reason might be that the market has become more and more unstable. Therefore, the structure of GARCH model might changes more frequently than four years and also more difficult to make accurate parameter estimation.

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