Index Development for a Market with Heavy-tailed Distributions

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Abstract

In order to fully reflect the movements of prices or returns on financial assets, their distributions should be considered. However, they are often heavy-tailed and possibly skewed, and identifying them directly is not easy. To address this problem, we shall propose a statistical method of constructing a price index of a financial asset where the price distributions are skewed and heavy-tailed. Firstly, the Box-Cox transformation is applied where the parameter is determined by minimizing the AIC with respect to the original data. Then, the long-term trend of the distributions is estimated by fitting a new trend model with time-varying observation noises. Finally, the index is defined by taking the inverse Box-Cox transformation of the optimal long-term trend. To show the effectiveness of our method, it will be applied to the sovereign Credit Default Swap market. The worldwide spillover effects of the European debt crisis will be detected. Applying our method to the markets with insufficient information such as fast-growing or immature markets can be effective.

Keywords: Box-Cox transformation, financial crisis, time series analysis, time-varying variance

1 Introduction

The heavy-tailed distributions have often been found in the prices or returns of financial markets and identifying them directly has not been easy. Since Mandelbrot (1963) and Fama (1965) studied that the asset return distribution has heavier tails than a normal distribution and can be better be described as a stable Paretian distribution, there have been a lot of investigations on return distribution alternatives such as Praetz (1972) and Madan and Seneta (1990).

On the other hand, due to the globalization of the financial system, even the price fluctuations of fast-growing financial markets have become influential on the world economy. In order to construct an index fully reflecting the movements of financial asset prices, their distributions should be considered. To address this problem, we shall propose a statistical method of constructing a price index of a financial asset where the price distributions are skewed and heavy-tailed. Firstly, the Box-Cox transformation is applied where the parameter is determined by minimizing the AIC with respect to the original data. Then, the long-term trend of the distributions is estimated by fitting a new trend model with time-varying observation noises. In Tanokura et al. (2012), we proposed the method based on the trend estimation with time-varying Cauchy observation noises. In this paper, we will furthermore improve it to the new trend estimation with time-varying Gaussian observation noises by introducing a time-varying variance model (Kitagawa (1987, 2010)). The estimation is performed by applying state space representation and the Kalman filter/smoother. Finally, the index is defined by taking the inverse Box-Cox transformation of the optimal long-term trend. To our knowledge, there have been few studies on estimating heavy-tailed distributions by using a variable transformation and the AIC.

To show the effectiveness of our method, it will be applied to the sovereign Credit Default Swap (CDS) market with heavy-tailed spread distributions, which has been highlighted since the European sovereign debt crisis triggered by the revelation of the Greece deficit in the fall 2009. The number of observations of sovereign CDS varies over time due to the immaturity. CDS is an over-the-counter contract which a buyer periodically pays to a seller a CDS spread quoted as annual rate until the maturity date of the contract or a credit event occurs whichever comes first, and the seller will make a payment on the occurrence of the credit event. As the spread of a sovereign CDS which deals with sovereign risk on a country's government bond can be regarded as the market evaluation on the credit risk of the country, the spread fluctuation has significantly affected the world economy.

We will construct five regional sovereign risk indices by applying our method to the spreads of 82 sovereign CDS issues. To investigate the European debt crisis contagion, we apply power contribution analysis which detects the multi-dimensional sources of fluctuations between regions in terms of frequency domain properties (Akaike (1968); Tanokura and Kitagawa (2004)) for the three periods of the post-Lehman shock, the post-Greece debt crisis and the current. As a result, the worldwide spillover effects of the European debt crisis will be detected. Applying our method to the markets with insufficient information such as fast-growing or immature markets can be effective.

This paper consists of four sections. Section 2 introduces our method of index construction. In Section 3, we construct five regional sovereign risk indices by our method. The spillover effects are explicitly detected by applying power contribution analysis. Finally, we present our conclusions in Section 4.

2 Method of index construction

We will briefly present our index construction method. Let $p_i(n)$ $(i = 1, \ldots, k(n);$ $n = 1, \ldots, T)$ denote prices of the comprising issues of a financial market with heavy-tailed price distributions at time n. $k(n)$ denotes the number of observations. We note that $k(n)$ varies over time and can be zero at certain times, and that $p_i(n)$ is positive. To make identifying the distributions easy, we consider the Box-Cox transformation (Box and Cox (1964)) of the prices:

$$q_{i,\lambda}(n) = h(\ p_i(n)\ ) = \begin{cases} \lambda^{-1}(\ p_i(n)^{\lambda} - 1) & \lambda \neq 0 \\ \log\ p_i(n) & \lambda = 0. \end{cases}$$

Then, the long-term trend of the mean $y_\lambda(n)$ of $q_{i,\lambda}(n)$ $(i = 1, \ldots, k(n))$ is estimated by fitting the following trend model:

$$\begin{aligned} \Delta^2 t_\lambda(n) &= v_\lambda(n), & v_\lambda(n) &\sim N(\ 0, \tau_\lambda^2\ ) \\ y_\lambda(n) &= t_\lambda(n) + w_\lambda(n), & w_\lambda(n) &\sim N(\ 0, \sigma_\lambda(n)^2/k(n)\ ), \end{aligned} \tag{1}$$

where $\Delta t_\lambda(n) = t_\lambda(n) - t_\lambda(n-1)$. The noise $v_\lambda(n)$ follows the normal distribution with the mean 0 and the unknown variance τ_λ^2, and so does the observation noise $w_\lambda(n)$ the normal distribution with the mean 0 and the unknown variance $\sigma_\lambda(n)^2/k(n)$ depending on the number of observations. $\sigma_\lambda(n)^2$ is estimated by the time-varying variance model which is equivalent to the stochastic volatility model (Kitagawa (1987, 2010)). As the trend model in (1) can be expressed as the state space model, for each λ the state is estimated by using Kalman filter and so are

Table 1: Country list and skewness(average) of CDS spreads for the 5 regions.

Region		Country	Skewness
Asia Pacific (AP)	15	Australia, Hong Kong, Japan, New Zealand, China, Fiji, Indonesia, Korea, Malaysia, Pakistan, Philippines, Sri Lanka, Taiwan, Thailand, Vietnam	1.796
Developed Europe (DE)	19	Denmark, Finland, Iceland, Ireland, Norway, Sweden, UK, Cyprus, Greece, Italy, Malta, Portugal, Spain, Austria, Belgium, France, Germany, Netherlands, Switzerland	2.357
Emerging Europe (EE)	17	Bulgaria, Croatia, Estonia, Hungary, Kazakhstan, Latvia, Lithuania, Macedonia, Poland, Romania, Russia, Serbia, Slovakia, Slovenia, Turkey, Ukraine, Czech Republic	1.596
Middle East /Africa (MA)	16	Angola, Bahrain, Egypt, Ghana, Iraq, Israel, Jordan, Lebanon, Morocco, Nigeria, Oman, Qatar, Saudi Arabia, South Africa, Tunisia, UAE	1.529
Latin America (LA)	15	Argentina, Brazil, Chile, Colombia, Costa Rica, El Salvador, Guatemala, Jamaica, Mexico, Dominican Rep, Panama, Peru, Uruguay, Venezuela, Trinidad & Tobago	3.604
Total	82		

Table 2: $\mathrm{AIC}_\lambda^{'}$ for major transformations for 5 regions.

λ	AP	EE	WE	MA	LA	Total
1	-27,580	-28,163	-28,501	-29,441	-25,005	-138,690
0.5	-29,342	-29,879	-35,093	-30,060	-26,557	-150,931
0	-30,902	-30,731	-37,625	-30,267	-27,619	-157,145
-0.5	-32,238	-30,922	-38,661	-30,530	-27,982	-160,333
-1	-31,662	-30,189	-35,537	-30,605	-26,599	-154,591

the parameters by maximum likelihood estimation. The missing observations are interpolated by a smoothing algorithm (Kitagawa and Gersch (1996); Kitagawa (2010)).

The optimal λ is selected by minimizing the following $\mathrm{AIC}_\lambda^{'}$ values. Let AIC_λ denote the AIC value of the trend model in (1). Then, $\mathrm{AIC}_\lambda^{'}$ which is modified to the AIC of the model for the original prices is expressed as

$$\mathrm{AIC}_\lambda^{'} = \mathrm{AIC}_\lambda - 2\sum_{n=1}^{T} \log \left| \frac{dh}{dz} \right|_{z=z_\lambda(n)}, \qquad (2)$$

where dh/dz is the Jacobian of the Box-Cox transformation (Kitagawa (2010)). Finally, the price index is defined by the inverse Box-Cox transformation of the long-term trend.

3 Application to the sovereign Credit Default Swap market

We construct five regional sovereign risk indices by applying our method. The daily composite spreads of the 82 USD-denominated sovereign CDS 5-year issues are used (Source: Markit). The period examined is from 9/11/2003 to 3/29/2013 (2,492 trading days). As shown in Table 1, based on the spread distributions, we classify 82 countries into five regions: Asia Pacific (AP), Developed Europe (DE), Emerging Europe (EE), Middle East/Africa (MA) and Latin America (LA).

Now we estimate the long-term trend for each region, supposing that a CDS spread follows the distribution specific to its region. Table 2 shows the $\mathrm{AIC}_\lambda^{'}$ in

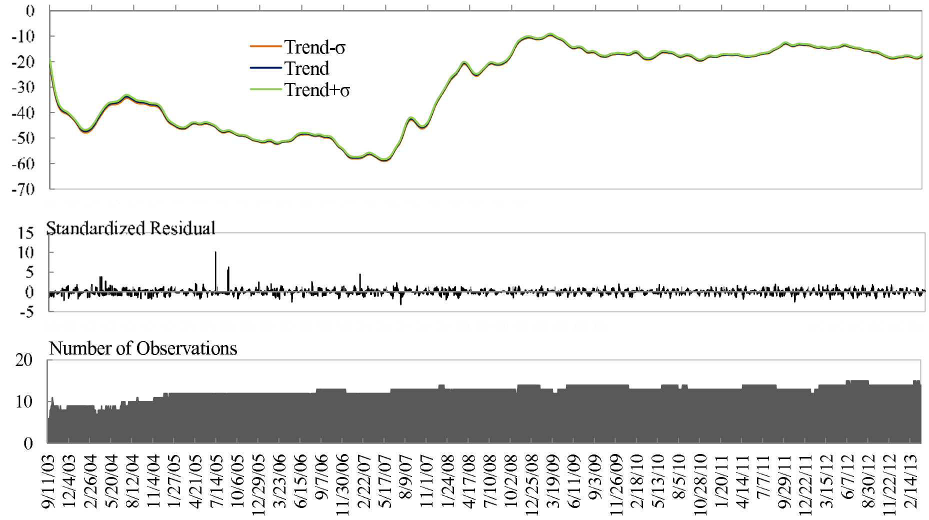

Figure 1: Estimated trend with the $\pm\sigma$, the standardized residual (the residual divided by the time-varying variance) and the number of observations for Asia Pacific (from top to bottom).

(2) for major transformations for five regions. It is noteworthy that λ with the minimum AIC'_λ is either -0.5 or -1. We determine the optimal λ as -0.5 for all regions. This transformation is the reciprocal square root when constant terms are ignored.

As an example, the long-term trend estimation for Asia Pacific is shown in Figure 1. Although the diversified residual terms can occasionally be seen in the cases of the sharp rise and fall of the spread, the trend is generally estimated well.

The relationships between the sovereign risk index and the price distributions for Developed Europe are shown in Figure 2. For simplicity, the data based on month-end are shown. The upside tails of the distributions distort after the Lehman shock in the fall of 2008, and significantly do again after the reveal of the Greece deficit in late 2009. The index (red-colored line) is mostly located close to the 50 percentile (dotted blue-colored one). Similar results are found for the other regions. As shown in Figure 3, the gradual uptrend of the DE index (blue-colored line) for the Post-Greece period can be found. Five regional sovereign risk indices are well-balanced to reflect the market views.

Finally, we investigate the spillover effects of the European sovereign debt crisis. We apply power contribution analysis (Akaike (1968); Tanokura and Kitagawa (2004)) to the 5-dimensional detrended sovereign risk indices, which are extracted by the program package DECOMP for three periods of the post-Lehman shock (9/15/2008 to 11/16/2009) the post-Greece crisis (11/17/2009 to 6/11/2012) and the current (6/12/2012 to 3/29/2013). It is developed by the Institute of Statistical Mathematics, based on the seasonal adjustment model (Gersch and Kitagawa (1983); Kitagawa and Gersch (1984)). Power contribution analysis measures the degree of multidimensional sources of fluctuations such as simultaneous influential components of various combinations of the noises in terms of frequency domain properties based on multivariate autoregressive (AR) modeling. See the details in Tanokura and Kitagawa (2004) and Tanokura et al. (2012).

Using the contribution score defined as aggregated power contribution % at

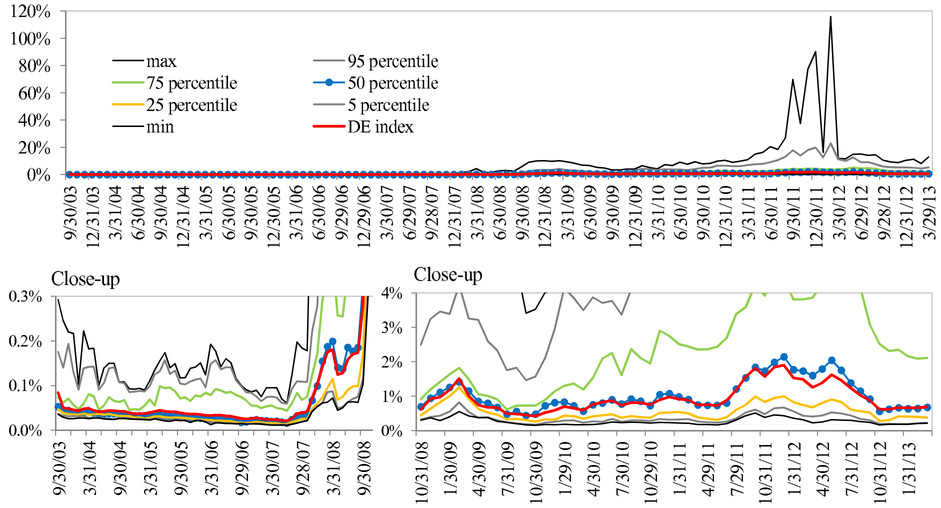

Figure 2: Developed Europe sovereign risk index (DE index) and the price distributions (the minimum, the 5, 25, 50, 75 and 95 percentiles, and the maximum) for Developed Europe on month-end base: the overview (top) and close-up for two sub-periods(bottom).

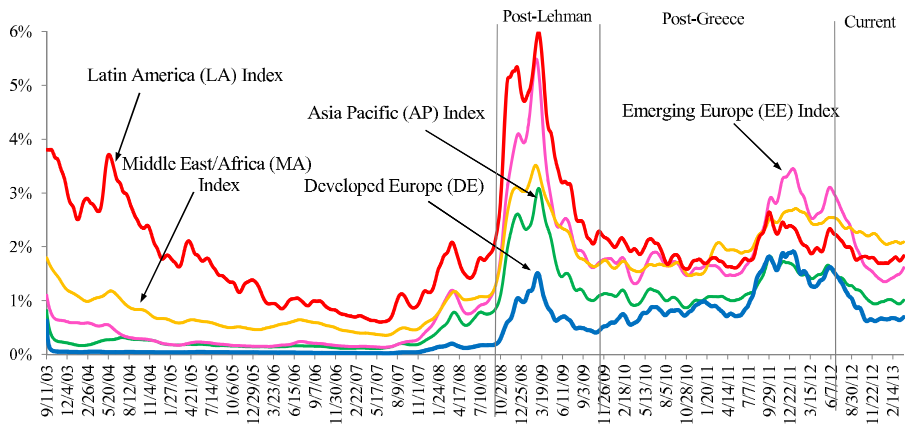

Figure 3: Five regional sovereign risk indices.

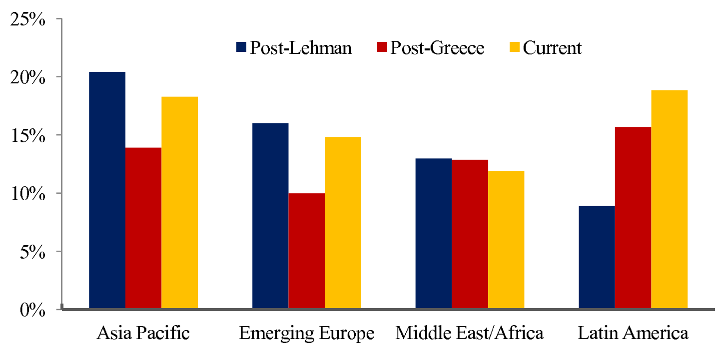

Figure 4: Contribution scores from Developed Europe to the other four regions for Post-Lehman, Post-Greece and the current periods.

the dominant frequency domain each region, Figure 4 focuses on the contribution scores from Developed Europe to the other four regions for three periods. The contribution scores for the current period become higher than those for the Post-Greece period for all regions excluding Middle East/Africa where those for all periods are almost the same level. This implies the worldwide spillover effects of the European debt crisis.

4 Conclusions

We presented a method of constructing an index where the price distributions are heavy-tailed. To show the effectiveness of our method, it was applied to the sovereign CDS market and the worldwide spillover effects of the European debt crisis was detected. Applying our method to the markets with insufficient information such as fast-growing or immature markets can be effective.

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