

Cross-sectional Vine Copula Factor Model

Mike K.P. So

Department of Information Systems, Business Statistics and Operations Management
The Hong Kong University of Science and Technology, Hong Kong immkps@ust.hk

Cherry Y.T. Yeung*

Department of Information Systems, Business Statistics and Operations Management
The Hong Kong University of Science and Technology, Hong Kong
cherryyeung032003@gmail.com

Vine decomposition on D-vines and canonical vines has been applied to estimate conditional dependence by expressing a high-dimensional copula function by a product of bivariate copulas and univariate distributions in vine-copula GARCH models. However, D-vines and canonical vines do not perform well for cross-sectional data such as financial stock returns. By incorporating market factors in the multivariate density, we propose a new cross-sectional vine copula model to make the copula decomposition parsimonious. The problem now concerns only the number of market factors, regardless of the dimension of financial returns. The main objective is to introduce a new vine decomposition for cross-sectional time series data which is computationally affordable for high dimensional problems (with a dimension of hundreds). Simulation study is performed to illustrate that our methodology works in high-dimensional situations. An empirical study with multiple financial time series is also conducted to illustrate this new vine decomposition.

Key words: Conditional dependence, vine decomposition, high dimensional problem, financial stock returns

1. Introduction

Most literature, such as Aas and Berg (2009); Austin and Lopes (2010); Dias and Embrechts (2010); Longin and Solnik (2001) and Patton (2006), in the context of pair copula copulas uses either canonical vines or D-vines to decompose a multivariate joint density into a product of marginal densities and conditional bivariate copulas. However, understanding of the dependence for cross-sectional data under canonical vines or D-vines is mysterious. Theoretical support is not given to ordering of variables and finding the most suitable tree structure. Gruber, Czado and Stöber (2012) proposed model selection on tree factorization, copula function and parameter estimations using MCMC methods. The computational burden for high dimensional problems, yet, still

needs more attention. In the aim to design a better pair copula construction for high dimensional cross-sectional data, we propose a factor copula model. This model defines relationship between return series and factors in the market through CAPM theory. Dependence is observed between return series and factors and among return series through commonly shared factors. By considering one factor case, single index market model (SIMM) assumes that each stock return induces systematic risk from only one factor in the market – market return. All the non-systematic risks are carried in the error term. SIMM model gives a crucial property that any return series is independent of other return series, conditional on the market factor. So when forming the dependence tree structure, there is only one layer.

Besides market return, stock returns may be dependent on other market factor such as inflation rate, interest rate etc. So we extend the factor model with multiple factors. Thanks to the crucial property of conditional independence in factor model, the number of tree level in the decomposition can be controlled and only relied on the number of factors in the models. Since usually small number of factors (less than ten) is sufficient in explaining stock returns, the consideration of ordering of variables is also easier than traditional canonical vines and D-vines methods as the dimensionality of the problem reduces to the number of factors instead of the number of stocks. This allows the model to analyze high dimensions problems, which are usually hundreds of stocks, at the same time. Time varying properties are incorporated to capture any impact on dependence level over time.

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