Spatial GARCH:
A spatial approach to multivariate volatility modelling

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This paper introduces a new approach to modelling the conditional variance in a multivariate setting. It is essentially a combination of the popular GARCH model class with a spatial component, inspired by generalized space-time models. The resulting spatial GARCH model takes into account both temporal and spatial dependencies in the conditional variance equation, by allowing the conditional variance of a particular market to depend on past volatility shocks in other markets. The inter-market dependencies are summarized in a weight matrix, which is specified beforehand using exogenous information such as market capitalization or countries' GDPs.

The main attractions of the spatial GARCH model are its simplicity, economic relevance, tractability and a low number of parameters, compared to other multivariate GARCH models. We specify the spatial GARCH model class, discuss the choice of the weight matrix, stationarity conditions and show how the model parameters can be estimated by the maximum likelihood method using an iterative GARCH maximum likelihood procedure.

We apply the spatial GARCH (1,1) model with various weight matrices to the returns from major stock markets. We find that the spatial GARCH (1,1) model is very well suited for modelling volatilities of an ensemble of stock markets, as the spatial parameters are highly significant. Moreover, spatial GARCH model excellently captures the high kurtosis present in squared returns, while regular GARCH fails to do so. Extending the model to incorporate leverage effects leads to further improvement in the volatility fit. We compare weight matrices based on the inverse travel distance, GDP and market capitalization and examine which weight matrix provides a more accurate description of spatial dependence.

Key Words: conditional variance, space-time models, GARCH, weight matrix, maximum likelihood estimation