Spatial GARCH: A spatial approach to multivariate volatility modelling

Svetlana Borovkova[1] and Rik Lopuhaa[2]
[1] Vrije Universiteit Amsterdam, The Netherlands
[2] Delft University of Technology, The Netherlands
[1] Corresponding author: Svetlana Borovkova, e-mail: s.a.borovkova@vu.nl

Abstract

This paper introduces a novel approach to modelling the conditional variance in a multivariate setting. It is a combination of the GARCH model and a “spatial” component, inspired by generalized space-time models. The inter-market dependencies are summarized in a weight matrix, which is specified beforehand using exogenous information.

The main attractions of the spatial GARCH model are its simplicity, economic relevance, tractability and a low number of parameters. We specify the spatial GARCH model class, discuss the choice of the weight matrix, stationarity conditions and the iterative maximum likelihood estimation method.

We apply the spatial GARCH (1,1) model to the returns from major stock markets and show that it is excellently suited for modelling volatilities of an ensemble of stock markets.

Keywords: conditional variance, space-time models, volatility spillover, GARCH, weight matrix, maximum likelihood estimation.

1. Introduction

Volatility is one of the most important concepts in finance. Estimating and modelling volatility is essential in the areas such as portfolio selection, risk management and option pricing. The seminal work by Engle (1982) introduced the so-called *autoregressive conditional heteroscedasticity model (ARCH)*, which, together with its many extensions, is now commonly used to model and forecast volatility of financial time series. The most important extension is the so-called *generalized ARCH*, or *GARCH model* by Bollerslev (1986). The low order GARCH model (GARCH (1,1)) gained a particular popularity, having just three parameters, while exhibiting a good performance for most financial series.

Extensions of a univariate GARCH model to the multivariate framework have the general name MGARCH (multivariate GARCH). Generally, these models attempt to describe the conditional dynamics of the entire variance-covariance matrix; the examples of such models are the VEC (vector autoregression-like specification of Bollerslev et al. (1988)) and BEKK model by Baba, Engle, Kraft and Kroner (1990) (see also Engle and Kroner (1995)). There are two main difficulties arising in the multivariate case: a large (and rapidly growing with the dimension) number of parameters and ensuring positive definiteness of the conditional covariance matrix. The curse of dimensionality has limited the application of unrestricted GARCH models to systems of higher dimensions. So many restricted versions of multivariate GARCH models have been introduced, the most famous being the Constant Conditional Correlation (CCC) and Dynamic Conditional Correlation (DCC) models.

Financial markets have become more interconnected over the past decade, as the recent financial crisis has shown. Already Bauwens et al. (2006) observe that volatilities move together across assets and markets. Shocks to volatility in one region often spill over to other regions and lead to increased volatility elsewhere. This is particularly pronounced for European countries, which are members of the same monetary union (the Eurozone), but is also observed outside of the EU. This motivates us to incorporate some spatial structure into a multivariate GARCH model. The space-time model class (known as STAR – Space-Time Autoregressive modes, widely applied in geology and ecology) is perfectly suited for this task, as this model class explicitly includes both temporal and spatial dependencies

2. Spatial GARCH model

A spatial GARCH model is essentially a combination of a GARCH model and a space-time model: it is obtained by adding a spatial component into the GARCH conditional variance equation.
Let $r_{t,i}$ be the logreturn of stock market *i* at time *t* with mean zero. Let random variables $Z_{t,i}$ be independent in *t* and identically distributed with zero mean, unit variance and constant covariance matrix *C*. Then we let $r_{t,i} = \sqrt{h_{t,i}} Z_{t,i}$, where $h_{t,i}$ is the conditional variance of the logreturns, which we write as

$$h_{t,i} = a_{0,i} + a_{1,i} r_{t-1,i}^2 + b_{1,i} h_{t-1,i} + a_{2,i} \sum_{j=1}^{N} w_{ij} r_{t-1,j}^2 + b_{2,i} \sum_{j=1}^{N} w_{ij} h_{t-1,j} , \quad (1)$$

where w_{ij} are the components of the spatial weight matrix, specified beforehand and satisfying the same conditions as above: $\sum_{j=1}^{N} w_{ij} = 1$ and $w_{ii} = 0$ for all i.
In the matrix notation, the spatial GARCH (1,1) model for an ensemble of *all N* stock markets' conditional variances can be written as follows:

$$H_t = A_0 + (A_1 + A_2 W) R_{t-1}^2 + (B_1 + B_2 W) H_{t-1}$$

where the vectors of conditional variances and squared logreturns are respectively $H_t = (h_{t,1},..., h_{t,N})^T$ and $R_t^2 = (r_{t,1}^2,..., r_{t,N}^2)^T$, A_0 is the parameter vector $A_0 = (a_{0,1},..., a_{0,N})^T$ and the *N x N* diagonal parameter matrices A_k and B_k are

$$A_k = \begin{pmatrix} a_{k,1} & 0 & 0 & 0 \\ 0 & a_{k,2} & 0 & 0 \\ 0 & 0 & ... & 0 \\ 0 & 0 & 0 & a_{k,N} \end{pmatrix}, \qquad B_k = \begin{pmatrix} b_{k,1} & 0 & 0 & 0 \\ 0 & b_{k,2} & 0 & 0 \\ 0 & 0 & ... & 0 \\ 0 & 0 & 0 & b_{k,N} \end{pmatrix}, \qquad k = 1,2.$$

The variance specification (1) models spillovers of volatilities across markets. Economically interpretable "spatial" dependencies between markets should be specified beforehand by means of the weight matrix *W*. The weight matrix should reflect interrelations between the markets or assets under consideration. If stock market volatilities of several countries are modelled, the weights should reflect linkages between the countries' economies; hence, it should be based on interpretable economic factors. The first, rather obvious choice is the weight matrix based on the inverse of the travel distances between stock market's cities. This choice can create the first impression of the spatial dependencies between the stock markets. Arguably, nearby countries have more economic and financial interconnections (due to historical and geographical linkages) and therefore their stock markets can have a greater influence on each other. This is especially the case in an economic and monetary union such as the Eurozone. Flavin et al. (2002) state that geographical variables matter when examining equity market linkages. They mention that, in particular, the number of overlapping opening hours and a common border tend to increase cross-country stock market correlations.

However, because of the increasing globalisation, it is not obvious whether distance is a major economic factor that contributes to spatial correlation. Globalisation can make other economic factors more relevant, for example, the market capitalization of the stock markets or the gross domestic product of the country in which the stock market is situated. It is likely that a larger market capitalization of a stock market leads to a greater influence on other stock markets; the same interpretation holds for GDP: a larger GDP is expected to lead to a more significant stock market which has more influence on other stock markets. The evidence for that is provided by e.g., Hanhardt and Ansotegui (2009), who document a significant influence of Germany (large GDP and large stock market capitalization) on all Eurozone stock markets.

The spatial GARCH model (1) can be estimated by means of the conditional MLE. For details of such estimation, as well as for stationarity conditions, we refer to our full paper. In the next section we will apply the spatial GARCH (1,1) model (1) to the major stock markets of Europe, US and UK and compare them to the collection of univariate (and unrelated) GARCH (1,1) models. We will use the inverse distance, GDP-based and market capitalization-based weight matrices and assess which one leads to a better volatility fit.

2. Results

The data consists of daily log-returns on blue-chip stock market indices of the initial Eurozone countries (excluding Luxembourg), UK and US, ranging from January 2002 until the end of 2010. The first weight matrix is based on the inverse travel distances between the stock market cities. The second weight matrix is based on GDPs of countries as of 2010, provided by the IMF. This matrix is more realistic, as US, UK and Germany get the highest weights, as we might expect. Finally, the market capitalization weight matrix is obtained from market capitalizations as of 2010, it is similar to the GDP weight matrix.

Figures 1 shows the parameter estimates for the spatial GARCH (1,1) model with the market capitalization weight matrices. Nearly all spatial parameters are highly significant. This indicates that the spatial volatility spillovers are indeed observed and significant among the considered stock markets. Spatial ARCH coefficients are generally larger than spatial GARCH ones, indicating that the latest squared returns from other markets matter more for the future volatility levels than the previous values of other markets' variances.

The spatial GARCH model leads to a dramatic improvement in the model fit, compared to the collection of univariate GARCH models (Table 1). Regarding the relative performance of various weight matrices, we can observe the following. First, the spatial GARCH models with three different weight matrices perform comparable to each other. For European countries, however, the model with the inverse distance weight matrix performs slightly better. The US stock market volatility is modelled equally well by all three weight matrices, while the case of UK lies somewhere in the middle. This indicates that, for the Eurozone, the geographic proximity is an important determinant of the stock markets interactions, as it reflects historically established economic links.

5. Conclusions

We presented the spatial GARCH model class, where we capture the market interdependencies and volatility spillovers by means of a pre-defined and economically relevant weight matrix. The model has a very low number of parameters: this number grows linearly with the dimension and not as the second (or even fourth) power of the dimension, as in other multivariate volatility models.

The application of the spatial GARCH model to an ensemble of major stock markets has shown that the spatial parameters are highly significant, indicating the presence of volatility spillovers between markets. The spatial GARCH model exhibits much better fit than the regular GARCH model; the fit is further improved by adding the component for the leverage effect. The model is capable to capture skewness and high kurtosis present in the data, in situations where the regular GARCH model fails to do so.

References

Bauwens L. S. , Laurent S. and Rombouts J.V.K. (2006) Multivariate GARCH Models: A Survey. *Journal of Applied Econometrics*, Vol 21, pp. 79-109.

Baba, Y., R. F. Engle, D. Kraft, and K. Kroner (1990) Multivariate Simultaneous Generalized ARCH, *UCSD, Department of Economics, unpublished manuscript*.

Bollerslev T. (1986) Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics,* Vol 31, pp. 207-327.

Bollerslev, T., R.F. Engle and J.M. Wooldridge (1988) A capital asset pricing model with time varying covariances. *Journal of Political Economy,* 96, pp. 116-131.

Borovkova S., Lopuhaä H.P., and Nurani B. (2008) Consistency and asymptotic normality of least squares estimators in generalized STAR models. *Statistica Neerlandica,* Vol 62, pp. 482-508.

Borovkova S., Lopuhaä H.P. (2012) Spatial GARCH model: A spatial approach to multivariate volatility modelling. Working paper, available on SSRN:

Engle, R. F. (1982) Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation, *Econometrica,* Vol 50 (4), pp. 987-1007.

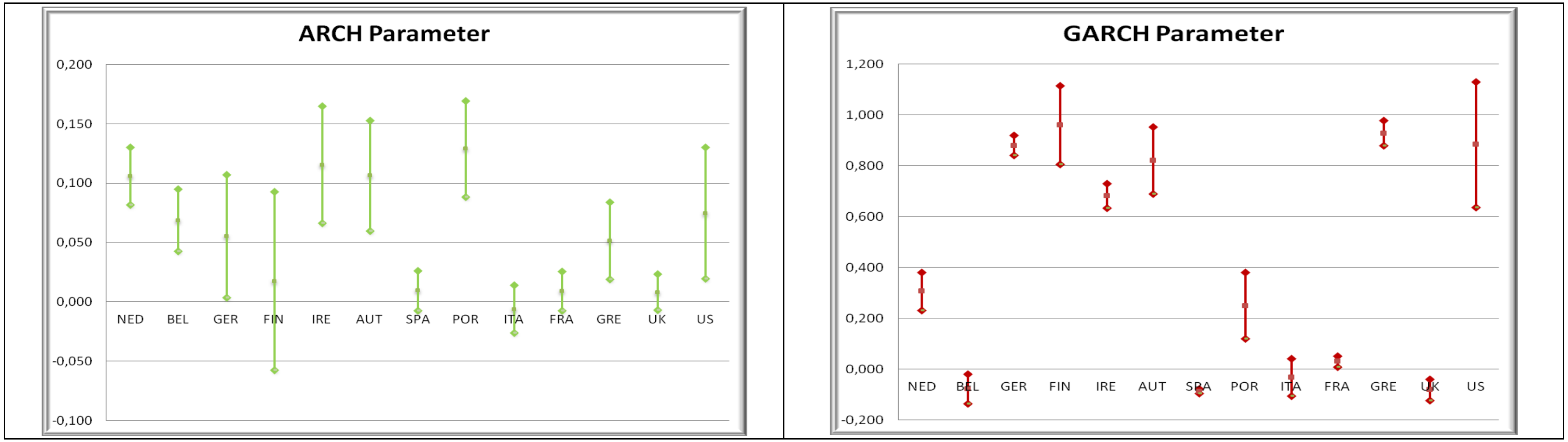
Engle R. F. and Kroner K.F. (1995) Multivariate Simultaneous Generalized ARCH. *Econometric Theory,* Vol 11, pp. 122-150.

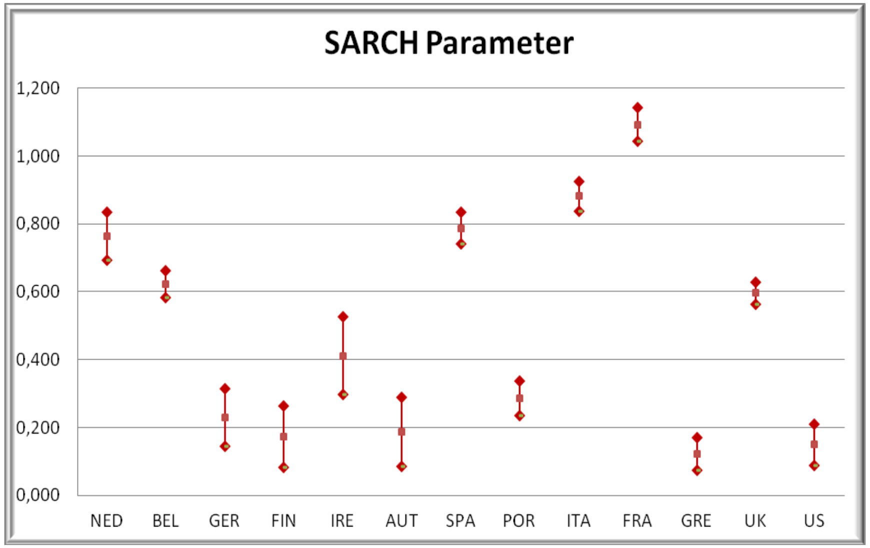
Engle, R.F. (2002) Dynamic conditional correlation: A simple class of multivariate GARCHmodels. *Journal of Business and Economic Statistics*, Vol. 20, pp. 339-350.

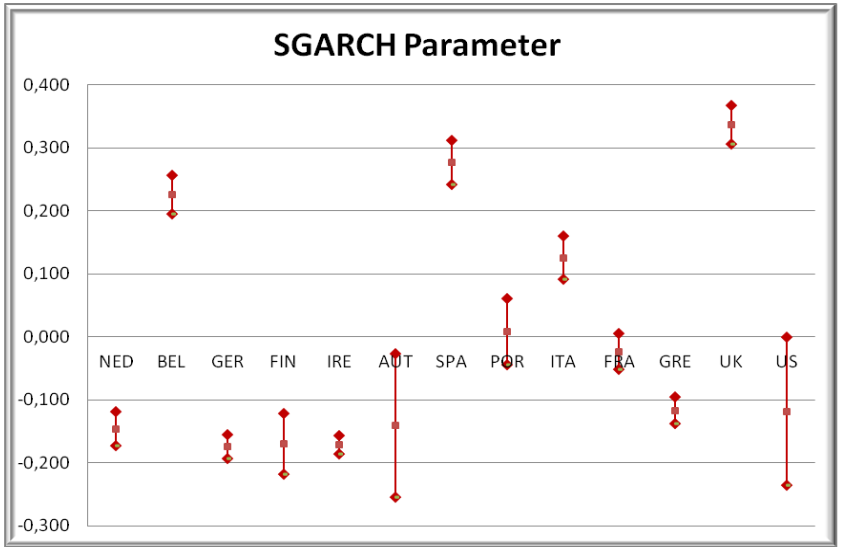
Flavin, T.J., M. J. Hurley and F. Rousseau (2002) Explaining Stock Market Corrlation: a Gravity Model Approach, *The Manchester School Supplement*, pp. 87-106.

Hanhardt, A. and Ansotegui, C. (2009) Measuring European Stock Market Integration Via a Stochastic Discount Factor Approach. *ESADE Business School, Barcelona, Spain.*

Figure 1: The estimated parameters with 95% confidence intervals for the spatial GARCH (1,1) model with the market capitalization weight matrix.







	AEX			FTSE 100			NYSE Composite		
GARCH (1,1)	4.80436E-07	6413	-12816	2.333E-07	6846	-13682	3.4969E-07	6812	-13614
Spatial GARCH (1,1)	3.15203E-07	6666	-13316	1.667E-07	7035	-14054	3.3735E-07	6865	-13713
SGJR-GARCH (1,1)	3.89229E-07	6580	-13140	1.661E-07	7039	-14058	3.2404E-07	6886	-13752

Table 1. Residual mean squared error (RMSE) and log-likelihood values for the AEX, FTSE 100 and NYSE Composite and market cap weight matrix.