Modeling inflation rates as long memory seasonal processes

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Abstract

In long memory time series, present values are strongly correlated with distant past. These series are stationary, although its autocorrelation functions are similar to nonstationary processes, which can lead to misspecified models. Long memory processes are especially useful in economics and finance. These processes are often modeled by ARFIMA models, which generalize ARIMA models by allowing non-integer values of the differencing parameter. Also, the series may present seasonal long memory. The so called SARFIMA models may handle with this pattern through a non-integer seasonal differencing parameter, but these kinds of models are not still so used. This paper aims to work with non-seasonal and seasonal long memory, through SARFIMA models. To do so, inflation rates of United States and United Kingdom are used, and forecasting results are used for comparison. SARFIMA and SARIMA models have similar performances, but SARFIMA models have the advantage of being more parsimonious, since the AR and MA coefficients are not necessary for good fitting and forecasting.

Keywords: long-term memory models, SARFIMA, inflation, forecasting.

1. Introduction

Long memory time series has been a topic of great interest by econometricians since around 1980, when some authors noted long term dependence in economic and financial data (Morettin and Toloi, 2006). However, long memory models have played a role in the physical sciences since at least 1950 (Baillie, 1996). A long memory time series is characterized by its autocorrelation function, which decays very slowly, in a hyperbolic way (Hosking, 1981). The autocorrelations may be significative during dozens or even hundreds of lags. Present values are strongly related with data from a very distant past – that's why "long memory".

The long memory time series autocorrelation function suggests that the series is nonstationary, which can lead to differencing it. However, the differenced series may seem overdifferencied (Morettin and Toloi, 2006). In fact, long memory time series tends to reject the existence of a unit root. ARFIMA modeling is the most used for long memory processes. They are a general form of ARIMA modeling. In ARFIMA processes, the differencing order is fractional and in general is a number between 0 and 0.5, which tells us that the series needs a differencing, but an integer differencing would seem excessive.

Many works have been done considering inflation rates as long memory series. Bos, Franses and Ooms (2002) show that ARFIMA modeling is quite useful in United States inflation rates. Baillie, Chung and Tieslau (1996) discover strong evidence of long memory in inflation rates of nine countries. Hassler and Wolters (1995) analyze inflation rates from five industrialized countries and reject unit root hypothesis. So they conclude that considering a fractionally difference order is reasonable.

The so-called SARFIMA models are more recent in literature. These models may present two fractionally differencing orders (non-seasonal and seasonal). Porter-Hudak (1990) applies SARFIMA modeling in monetary aggregates. Bisognin (2003) compares several estimation techniques through simulated data.

This paper is concerned with SARFIMA modeling in United States and United Kingdom inflation rates. The main concern of this work is the quest for more parsimonious models comparing to the ARIMA modeling, with similar or even better predictive performances.

2. Fractionally Differenced Models

An ARFIMA process is a generalization of an ARIMA process. In ARFIMA, the integration order d may be a non-integer number. A X_t process is an autoregressive fractionally differenced moving average, ARFIMA(p,d,q), if X_t is stationary and satisfies the equation

$$\varphi(B)(1-B)^d X_t = \theta(B)a_t \tag{1}$$

where a_t is a white-noise process, $\varphi(B)$ and $\theta(B)$ are autoregressive and moving average polynomials with orders p and q, and d is the fractionally differenced component which lies inside the interval (–0.5, 0.5). The fractionally differencing operator $(1 – B)^d$ is defined by

$$(1-B)^d = 1 - dB - \frac{d(1-d)}{2!}B^2 - \frac{d(1-d)(2-d)}{3!}B^3 - \cdots \tag{2}$$

for any real number $d > –1$. The effect of d decays hiperbolically as the time increases. For more details, please check Beran (1994) and Morettin and Toloi (2006).

We are interested in $d \in (0, 0.5)$. When $d < 0$, it can't be interpreted as a fractionally differencing order in the sense we are used to. When $d \geq 0.5$, the process is nonstationary (Beran, 1994). The ARFIMA process can be extended to contemplate the seasonal behavior of the data:

$$\varphi(B)\Phi(B^s)(1-B)^D(1-B)^d X_t = \theta(B)\Theta(B^s)a_t \tag{3}$$

where Φ and Θ are the seasonal autoregressive and moving average polynomials with P and Q orders, and s is the seasonal period. The seasonal fractionally differencing operator $(1 – B^s)^D$ is defined by

$$(1-B^s)^D = 1 - DB^s - \frac{D(1-D)}{2!}B^{2s} - \frac{D(1-D)(2-D)}{3!}B^{3s} - \cdots \tag{4}$$

for any real number $D > –1$. This is the SARFIMA$(p,d,q)\times(P,D,Q)_s$ process. The development of the expressions (2) and (4) leads to

$$(1-B)^d(1-B^s)^D X_t = \left(1 - \sum_{i=1}^{\infty} \lambda_i B^j\right) X_t \tag{5}$$

where

$$\lambda_{ms+k} = \frac{\prod_{j=0}^{ms+k-1}(j-d)}{(ms+k)!} + \sum_{i=1}^{m} \frac{\prod_{j=0}^{(m-i)s+k-1}(j-d)}{((m-i)s+k)!} \frac{\prod_{j=0}^{i-1}(j-D)}{i!} \tag{6}$$

In this paper, the parameters d and D are estimated in an empirical way. They are chosen so that the differenced series, resulting of the expression above, is the more close to a white-noise.

3. Data

United States (January/1957 to December/2010) and United Kingdom (January/1950 to December/2010) CPI (Consumer Price Index) inflation rates are used in this paper. Data from January to December/2011 will be used for prediction comparisons. United

States CPI data can be found at IMF (International Monetary Fund) website. United Kingdom data are available at ONS (Office for National Statistics). Since inflation is a measure of prices variation, Baillie (1996) suggests taking the first difference of CPI logarithm, which explains the prices variations in an additive and non-cumulative fashion. Data and its autocorrelation functions are shown in Figures 1 and 2.

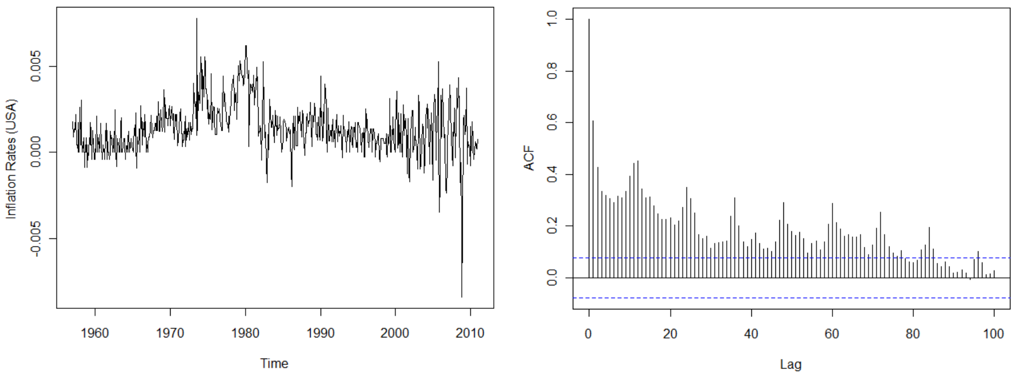

Figure 1: Inflation rates for USA: (a) Time series plot; and
(b) Sample autocorrelation function

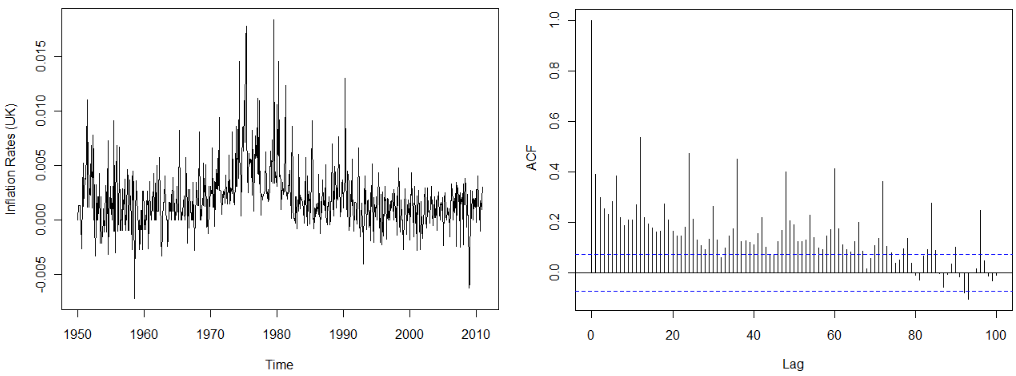

Figure 1: Inflation rates for UK: (a) Time series plot; and
(b) Sample autocorrelation function

The autocorrelation functions decay very slowly for both series, a phenomenon similar to the autocorrelation function of nonstationary time series (Wei, 2006). The existence of non-seasonal and seasonal unit roots is tested by *R* functions *ndiffs* and *nsdiffs*. The results are presented in Table 1. The existence of unit roots is rejected for United States and United Kingdom. As both non-seasonal and seasonal autocorrelations are persistent, a SARFIMA modeling seems reasonable.

Before SARFIMA fiting, the airline model SARIMA(0,1,1)×(0,1,1) is fitted for both series, just for residuals outliers detection. The airline model is a very famous one and it offers a good fitting for several time series (Box and Jenkins, 1970). In fact, it provides residuals close to white-noise for both series. Hair et al (2010) suggest to remove or replace observations which leads to residuals bigger than 4 or less than –4 in big samples. In United States data, five observations were replaced by its SARIMA fitted values. In United Kingdom, four observations were replaced.

Table 1: Unit root tests p-values

Test	**USA**	**UK**
Phillips-Perron (non-seasonal)	<0.05	<0.05
OCSB (seasonal)	<0.05	<0.05

4. Results

A function was written in *R* to evaluate the series after non-seasonal and seasonal fractionally differencing. Values of *d* and *D* between 0 and 0.5 were considered, in a 0.001 grid. For each combination of *d* and *D*, the sum of the significative autocorrelations was computed for the differenced series, from lags 1 to 100. The optimal values of *d* and *D*, which can be seen in Table 2, were chosen so this sum is the smallest possible, which represents the closest to a white-noise.

Table 2: Estimated values of fractional orders

Fractional order	**USA**	**UK**
d (non-seasonal)	0.367	0.196
D (seasonal)	0.208	0.314

Figure 3 presents the autocorrelation function for the United States differenced series, with *d* = 0.367 and some values of *D*. It is worth to note that, when *D* is bigger than 0.2, the number of significative autocorrelations increases, which suggests that a higher differencing order is not necessary. In fact, when a integer differencing order is taken, the autocorrelation function suggests an MA(1) seasonal process, due to a high autocorrelation at lag 12. So, it would be necessary to estimate the differencing order (which would be 1) and a moving average parameter. The same pattern would be noted in the case of a non-seasonal integer differencing (a MA(1) process), which would lead to the airline model.

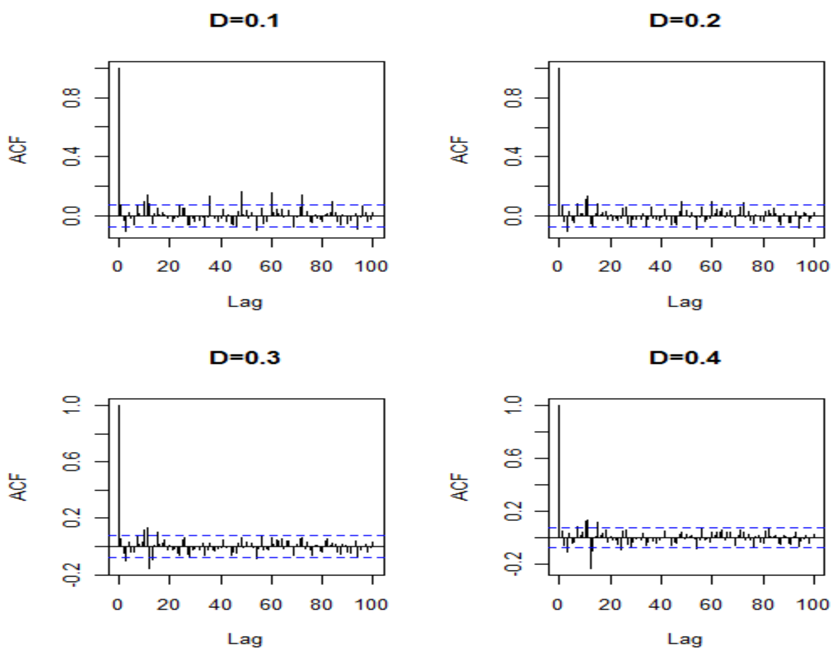

Figure 3: Sample autocorrelation function of USA inflation rates
for some values of *D* and *d* = 0.367 fixed

Figures 4 and 5 show the residual autocorrelation function resulting of both SARIMA and SARFIMA modeling. The observed pattern is similar to a white-noise process. Since there are more than 600 observations in both series, tests tend to reject the normality of residuals. So, to check normality, samples of size 100 were taken, and in 70-80% of the samples, Shapiro-Wilk test would not reject the normality hypothesis.

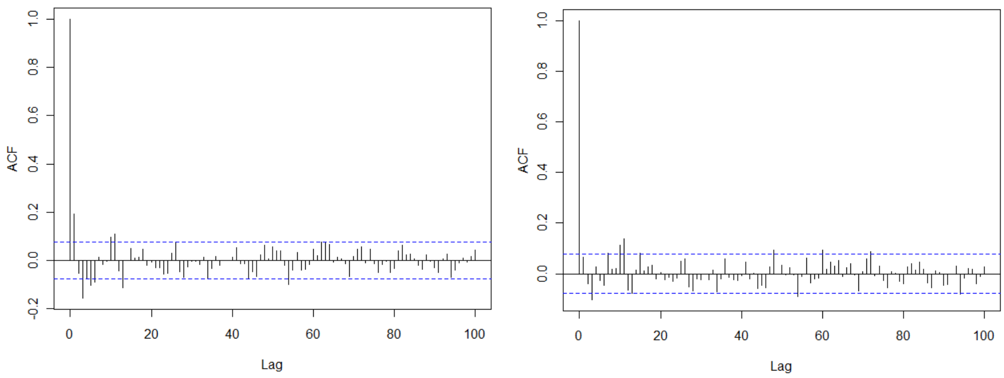

Figure 4: Sample autocorrelation function for United States inflation rates:
(a) SARIMA (b) SARFIMA

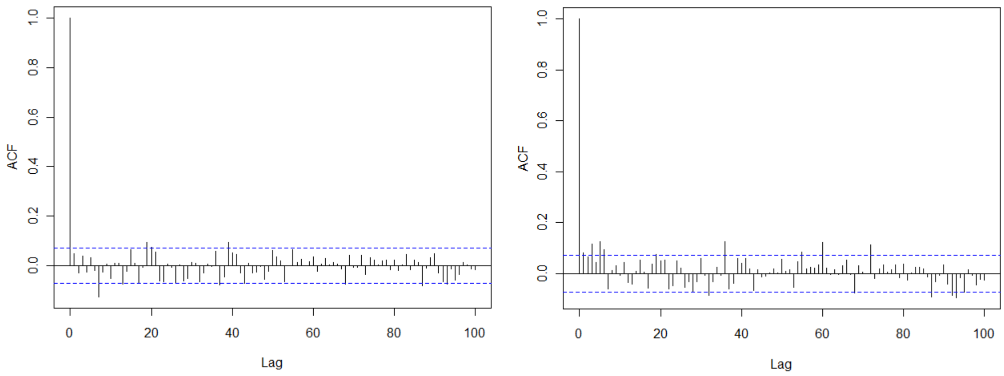

Figure 5: Sample autocorrelation function for United Kingdom inflation rates:
(a) SARIMA (b) SARFIMA

For comparison purposes, the predicted values from January to December/2011 are computed, for both series and both models. Using real observed values, the Mean Absolute Error (MAE) and Mean Squared Error (MSE) are computed for three, six and twelve months forecasting horizons and are shown in Tables 3 and 4.

Table 3: United States inflation rates: predictive comparison

	MAE		
Model	**3 months**	**6 months**	**12 months**
SARIMA	$7{,}37\times10^{-4}$	$9{,}54\times10^{-4}$	$8{,}10\times10^{-4}$
SARFIMA	$1{,}36\times10^{-3}$	$1{,}37\times10^{-3}$	$1{,}03\times10^{-3}$
	MSE		
Model	**3 months**	**6 months**	**12 months**
SARIMA	$1{,}06\times10^{-6}$	$1{,}40\times10^{-6}$	$1{,}01\times10^{-6}$
SARFIMA	$2{,}66\times10^{-6}$	$2{,}31\times10^{-6}$	$1{,}53\times10^{-6}$

Table 4: United Kingdom inflation rates: predictive comparison

	MAE		
Model	**3 months**	**6 months**	**12 months**
SARIMA	$1{,}21\times10^{-3}$	$1{,}28\times10^{-3}$	$9{,}92\times10^{-4}$
SARFIMA	$1{,}17\times10^{-3}$	$9{,}06\times10^{-4}$	$8{,}22\times10^{-4}$
	MSE		
Model	**3 months**	**6 months**	**12 months**
SARIMA	$2{,}02\times10^{-6}$	$2{,}23\times10^{-6}$	$1{,}47\times10^{-6}$
SARFIMA	$1{,}88\times10^{-6}$	$1{,}27\times10^{-6}$	$1{,}05\times10^{-6}$

For United Kingdom CPI inflation rates, the SARFIMA model provides a better set of predicted values, for all prediction horizons. For United States, SARIMA model have a slight advantage, but it is worth to note that SARFIMA model has only two parameters (*d* and *D*), instead of four used in SARIMA (*d*, *D* and the moving average parameters).

5. Conclusions

SARFIMA modeling is successful when applied in United States and United Kingdom CPI inflation rates. When fractionally differencing orders are considered, additional autoregressive and moving average parameters (which might be needed in models with integer differencing orders) may not be necessary. The residuals resulting from SARFIMA modeling are reasonable with the assumptions of non-correlation and normality. In terms of forecasting, SARFIMA model is better than airline SARIMA model for United Kingdom series and it has a slight disadvantage for United States series. The main contribution is that SARFIMA model can reach similar or even better results than SARIMA through a simpler model, with only two parameters needed – the non-seasonal and seasonal fractionally differencing orders.

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