Reliability and Profit Analysis of a Computer System with Hardware Repair and Software Replacement Subject to Conditional Arrival Time of Server

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Abstract

The focus of this paper is on reliability and profit analysis of a two unit cold standby computer system. In each unit h/w and s/w fail independently from normal mode. There is a single server who takes some time to arrive at s/w failure with the condition that he has to visit the system immediately at h/w failure. The unit is repaired at h/w failure while s/w is replaced by new one with some replacement time when it fails to execute the required instructions properly. Server can leave the system only after finishing all jobs available to him. All random variables are statistically independent. The time to failure of h/w and s/w is distributed exponentially while the arrival time of the server, h/w repair time and s/w replacement time are arbitrarily distributed with different probability density functions (pdf). The expressions for various reliability measures are derived using semi-Markov process and regenerative point technique. The results for a particular case are obtained to depict the graphical behaviour of some important measures of system effectiveness.

Keywords: Computer System, H/w repair, S/w Replacement, Conditional Arrival Time of the Server, Reliability and Profit Analysis.

Subject Classification: Primary 90B25 and Secondary 60K110.

1. Introduction

In the modern society, it is hard to imagine any area in which computer systems do not play a dominant role. Computers are being used frequently in most of the critical areas such as aerospace, nuclear power generation and defence. For such applications their reliability is the upmost importance because a computer failure in these areas could be very costly and dangerous to the society. Other factors such as increasing repair costs, harsher operating environments, use by novices and the existence of bigger systems are also responsible for the increasing emphasis on reliability of computer systems. However, in computers the reliability problem is not only confined to the h/w aspect but also extends to s/w. Both h/w and s/w have to be reliable for successful operation of a computer. Therefore, there is a definite need to place emphasis on the reliability of both the computer h/w and s/w. The method of redundancy is one of the best techniques to improve the reliability of any operating systems. Therefore, in recent years, reliability models of two-unit cold standby computer systems having independent h/w and s/w failures have been suggested by the researchers including Malik and Anand [2010 and 2012]. Also, Malik and Sureria [2012] analyzed reliability models for a cold standby computer system with h/w repair and s/w replacement by a server who visits the system immediately whenever needed. But, sometimes, it becomes difficult for a server to attend the system promptly when needed may because of his pre-occupations. And, in such a situation, the server may take some time to arrive at the system (called arrival time of the server) with the condition that he has to attend immediately the serious technical hitches occurred in the system.

The motive of the present study is to make reliability and profit analysis of a computer system by taking two identical units of it- one is initially operative and the other is kept as spare in cold standby. The computer is considered as a single unit in which h/w and s/w work together and may have independent failure from normal mode. There is a single server who is allowed to take some time at s/w failure with the condition that he has to visit the system immediately at h/w failure. The repair of the unit is done at h/w failure while s/w is replaced by new one giving some replacement time when it fails to execute the required instructions properly. Server can leave the system only after finishing all the jobs available to him. The random variables are statistically independent. The distribution of failure time of h/w and s/w is taken as negative exponential whereas h/w repair time, s/w replacement time and arrival time of the server are arbitrarily distributed. To meet out the objectives, the expressions for various reliability measures such as mean sojourn times, mean time to system failure (MTSF), availability, busy period of the server due to hardware and software failures, expected number of replacements of the software, expected number of visits by the server and profit are derived using semi-Markov process and regenerative point technique. The graphical study of the results has also been made for a particular case.

2. Notations

E	:	The set of regenerative states
O	:	The unit is operative and in normal mode
Cs	:	The unit is cold standby
a/b	:	Probability that the system has hardware / software failure

λ_1/λ_2 : Constant hardware / software failure rate

FHUr/FHUR : The unit is failed due to hardware and is under repair / under repair continuously from previous state

FHWr / FHWR : The unit is failed due to hardware and is waiting for repair/ waiting for repair continuously from previous state

FSURp/FSURP : The unit is failed due to the software and is under replacement/ under replacement continuously from previous state

FSWRp/FSWRP : The unit is failed due to the software and is waiting for replacement / waiting for replacement continuously from previous state

w(t) / W(t) : pdf / cdf of arrival time of server due to s/w failure

f(t) / F(t) : pdf / cdf of replacement time of the software

g(t) / G(t) : pdf / cdf of repair time of the unit due to hardware failure

q_{ij} (t)/ Q_{ij}(t) : pdf / cdf of passage time from regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in (0, t]

$q_{ij.kr}$ (t)/$Q_{ij.kr}$(t) : pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k, r once in (0, t]

m_{ij} : Contribution to mean sojourn time (μ_i) in state S_i when system transits directly to state S_j so that $\mu_i = \sum_j m_{ij}$ and $m_{ij} = \int t dQ_{ij}(t) = -q_{ij}^{*}{}'(0)$

Ⓢ/© : Symbol for Laplace-Stieltjes convolution/Laplace convolution

~ / * : Symbol for Laplace Steiltjes Transform (LST) / Laplace Transform (LT)

' (desh) : Used to represent alternative result

The following are the possible transition states of the system:

S_0= (O, Cs), S_1= (O, FHUr),S_2= (O, FSWRp), S_3= (O, FSURp),S_4= (FHWr, FSURP),S_5= (FSWRP, FSWRp),

S_6= (FHUr, FSWRP),S_7 = (FHUR, FSWRp),S_8 = (FHUR, FHWr),S_9= (FSURp, FSWRP),S_{10}= (FSURP, FSWRp)

The state S_0–S_3 is regenerative states while the states S_4–S_{10} are non-regenerative as shown in figure 1.

State Transition Diagram

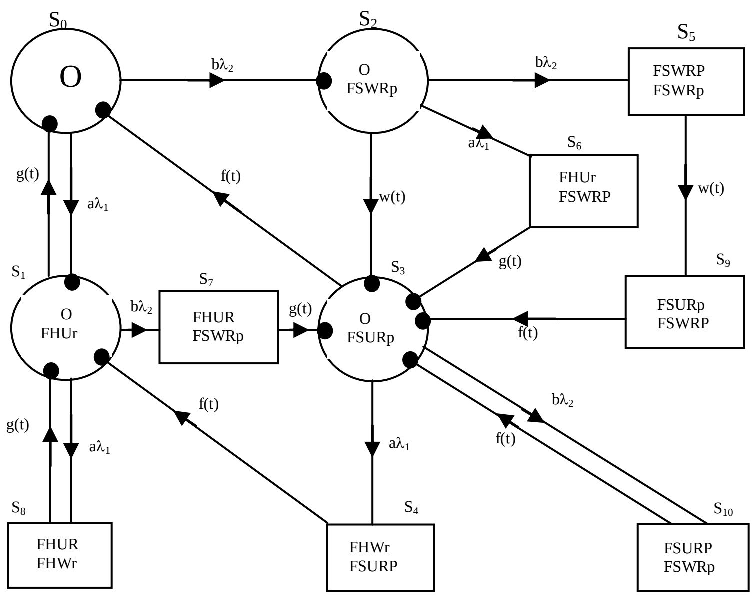

Fig. 1

○ Up-state □ Failed state ● Regenerative point

3. Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$p_{ij} = Q_{ij}(\infty) = \int_0^{\infty} q_{ij}(t)dt \quad \text{as}$$

$p_{01}=\dfrac{a\lambda_1}{a\lambda_1+b\lambda_2}$, $p_{02}=\dfrac{b\lambda_2}{a\lambda_1+b\lambda_2}$, $p_{10}=g^*(a\lambda_1+b\lambda_2)$, $p_{17}=\dfrac{b\lambda_2}{a\lambda_1+b\lambda_2}\left[1-g^*(a\lambda_1+b\lambda_2)\right]$,

$p_{18}=\dfrac{a\lambda_1}{a\lambda_1+b\lambda_2}\left[1-g^*(a\lambda_1+b\lambda_2)\right]$, $p_{23}=w^*(a\lambda_1+b\lambda_2)$, $p_{25}=\dfrac{b\lambda_2}{a\lambda_1+b\lambda_2}\left[1-w^*(a\lambda_1+b\lambda_2)\right]$,

$p_{26}=\dfrac{a\lambda_1}{a\lambda_1+b\lambda_2}\left[1-w^*(a\lambda_1+b\lambda_2)\right]$, $p_{30}=f^*(a\lambda_1+b\lambda_2)$, $p_{34}=\dfrac{a\lambda_1}{a\lambda_1+b\lambda_2}\left[1-f^*(a\lambda_1+b\lambda_2)\right]$,

$$p_{3,10}=\frac{b\lambda_2}{a\lambda_1+b\lambda_2}\left[1-f^*(a\lambda_1+b\lambda_2)\right] \quad (1)$$

For $f(t)=\theta e^{-\theta t}$, $g(t)=\alpha e^{-\alpha t}$ and $w(t)=\beta e^{-\beta t}$ we have

$p_{13.7}=\dfrac{b\lambda_2}{a\lambda_1+b\lambda_2+\alpha}$, $p_{11.8}=\dfrac{a\lambda_1}{a\lambda_1+b\lambda_2+\alpha}$, $p_{23.59}=\dfrac{b\lambda_2}{a\lambda_1+b\lambda_2+\beta}$, $p_{23.6}=\dfrac{a\lambda_1}{a\lambda_1+b\lambda_2+\beta}$,

$$p_{31.4}=\frac{a\lambda_1}{a\lambda_1+b\lambda_2+\theta},\quad p_{33.10}=\frac{b\lambda_2}{a\lambda_1+b\lambda_2+\theta} \quad (2)$$

It can be easily verified that $p_{01}+p_{02}=p_{10}+p_{17}+p_{18}=p_{23}+p_{25}+p_{26}=p_{30}+p_{34}+p_{3,10}=p_{10}+p_{11.8}+p_{13.7}$

$=p_{23}+p_{23.59}+p_{23.6}=p_{30}+p_{31.4}+p_{33.10}=1$ (3)

The mean sojourn times (μ_i) is the state S_i are

$$\mu_0=\frac{1}{a\lambda_1+b\lambda_2},\ \mu_1=\frac{1}{a\lambda_1+b\lambda_2+\alpha},\ \mu_2=\frac{1}{a\lambda_1+b\lambda_2+\beta},\ \mu_3=\frac{1}{a\lambda_1+b\lambda_2+\theta} \quad (4)$$

also

$$m_{01}+m_{02}=\mu_0,\ m_{10}+m_{17}+m_{18}=\mu_1,\ m_{23}+m_{25}+m_{26}=\mu_2,\ m_{30}+m_{34}+m_{3,10}=\mu_3 \quad (5)$$

and

$$m_{10}+m_{11.8}+m_{13.7}=\mu_1',\ m_{20}+m_{23.6}+m_{23.56}=\mu_2',\ m_{30}+m_{31.4}+m_{33.10}=\mu_3'\text{(say)} \quad (6)$$

For $f(t)=\theta e^{-\theta t}$, $g(t)=\alpha e^{-\alpha t}$ and $w(t)=\beta e^{-\beta t}$

we have $\mu_1^1=\dfrac{1}{\alpha}$, $\mu_2'=\dfrac{(\alpha+a\lambda_1)\theta\beta+b\lambda_2\alpha(\beta+\theta)}{\alpha\theta\beta(a\lambda_1+b\lambda_2+\beta)}$, $\mu_3^1=\dfrac{1}{\theta}$ (7)

4. Reliability and Mean Time to System Failure (MTSF)

Let $\phi_i(t)$ be the c.d.f. of first passage time from regenerative state i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$:

$$\phi_i(t)=\sum_j Q_{i,j}(t)(S)\phi_j(t)+\sum_k Q_{i,k}(t) \quad (8)$$

where *j* is an un-failed regenerative state to which the given regenerative state *i* can transit and *k* is a failed state to which the state *i* can transit directly. Taking LST of above relation (8) and solving for $\tilde{\phi}_0(s)$

We have

$$R^*(s)=\frac{1-\tilde{\phi}_0(s)}{s} \quad (9)$$

The reliability of the system model can be obtained by taking Laplace inverse transform of (9).

The mean time to system failure (MTSF) is given by

$$\text{MTSF}=\lim_{s\to o}\frac{1-\tilde{\phi}_0(s)}{s}=\frac{N_1}{D_1},\text{ where} \quad (10)$$

$N_1=\mu_0+p_{01}\mu_1+p_{02}\mu_2+p_{02}p_{23}\mu_3$ and $D_1=1-p_{01}p_{10}-p_{02}p_{23}p_{30}$

5. Steady State Availability

Let A_i (t) be the probability that the system is in up-state at instant‘t’ given that the system entered regenerative state i at t = 0. The recursive relations for A_i (t) are given as

$$A_i(t) = M_i(t) + \sum_j q_{i,j}^{(n)}(t) © A_j(t) \qquad (11)$$

where *j* is any successive regenerative state to which the regenerative state *i* can transit through n transitions and M_i (t) is the probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state, we have

$$M_0(t) = e^{-(a\lambda_1+b\lambda_2)t}, M_1(t) = e^{-(a\lambda_1+b\lambda_2)t}\bar{G}(t), M_2(t) = e^{-(a\lambda_1+b\lambda_2)t}\overline{W}(t), M_3(t) = e^{-(a\lambda_1+b\lambda_2)t}\bar{F}(t) \qquad (12)$$

Taking LT of above relations (11) and solving for $A_0^*(s)$, the steady state availability is given by

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2}, \text{ where} \qquad (13)$$

$N_2 = [p_{30}(1-p_{11.8}) + p_{10}\, p_{31.4}]\ \mu_0 + [p_{31.4} + p_{01}p_{30}]\ \mu_1 + p_{02}\ [p_{30}(1-p_{11.8}) + p_{10}\, p_{31.4}]\ \mu_2 + [p_{31.7} + p_{02}p_{10}]\ \mu_3$

$D_2 = [p_{30}(1-p_{11.8}) + p_{10}\, p_{31.4}]\ \mu_0 + [p_{31.4} + p_{01}p_{30}]\ \mu_1' + p_{02}\ [p_{30}(1-p_{11.8}) + p_{10}\, p_{31.4}]\ \mu_2' + [p_{31.7} + p_{02}p_{10}]\ \mu_3'$

6. Busy Period Analysis for Server

(a) Due to Hardware Repair

Let $B_i^H(t)$ be the probability that the server is busy in repairing the unit due to hardware failure at an instant ‘t’ given that the system entered state i at t = 0. The recursive relations $B_i^H(t)$ for are as follows:

$$B_i^H(t) = \mathrm{W_i}^{\mathrm{H}}(\mathrm{t}) + \sum_j q_{i,j}^{(n)}(t) © B_j^H(t) \qquad (14)$$

where *j* is any successive regenerative state to which the regenerative state *i* can transit through n transitions and W_i^H (t) be the probability that the server is busy in state S_i due to hardware failure up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_1^H(t) = e^{-(a\lambda_1+b\lambda_2)t}\bar{G}(t) + \left[a\lambda_1 e^{-(a\lambda_1+b\lambda_2)} © 1\right]\bar{G}(t) + \left[b\lambda_2 e^{-(a\lambda_1+b\lambda_2)t} © 1\right]\bar{G}(t) \qquad (15)$$

(b) Due to Replacement of the Software

Let B_i^S (t)be the probability that the server is busy due to replacement of the software at an instant ‘t’ given that the system entered the regenerative state i at t = 0. We have the following recursive relations for B_i^S (t):

$$B_i^S(t) = \mathrm{W_i}^S(\mathrm{t}) + \sum_j q_{i,j}^{(n)}(t) © B_j^S(t) \qquad (16)$$

where *j* is any successive regenerative state to which the regenerative state *i* can transit through n transitions and W_i^S (t) be the probability that the server is busy in state S_i due to replacement of the software up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_3^S(t) = e^{-(a\lambda_1+b\lambda_2)t}\overline{F}(t) + (a\lambda_1 e^{-(a\lambda_1+b\lambda_2)t} © 1)\overline{\mathrm{F}}(t) + (b\lambda_2 e^{-(a\lambda_1+b\lambda_2)t} © 1)\overline{\mathrm{F}}(t) \qquad (17)$$

Taking LT of above relations (14) and (16) and solving for B_0^{*H} (s) and B_0^{*S} (s), the time for which server is busy due to repair and replacements respectively is given by

$$B_0^H = \lim_{s\to 0} sB_0^{*H}(s) = \frac{N_3^H}{D_2} \text{ and} \qquad (18)$$

$$B_0^S = \lim_{s\to 0} sB_0^{*S}(s) = \frac{N_3^S}{D_2}, \text{ where} \qquad (19)$$

$N_3^H = [p_{01}(1-p_{33.10}) + p_{02}p_{31.4}]\tilde{W}_1^H(0)$, $N_3^S = [p_{01}p_{31.4} + p_{02}p_{11.8}]\tilde{W}_3^S(0)$ and D_2 is already mentioned.

7. Expected Number of Replacements of the Software

Let $R_i^S(t)$ be the expected number of replacements of the failed software by the server in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for $R_i(t)$ are given as

$$R_i^S(t) = \sum_j Q_{i,j}^{(n)}(t)(S)\left[\delta_j + R_j^S(t)\right] \quad (20)$$

Where j is any regenerative state to which the given regenerative state *i* transits and δj =1, if *j* is the regenerative state where the server does job afresh, otherwise δj = 0.Taking LST of relations (20) and solving for $\tilde{R}_0(s)$. The expected number of replacements per unit time to the software failures is given by

$$R_0(\infty) = \lim_{s\to 0} s\tilde{R}_0^S(s) = \frac{N_4}{D_2}, \text{ where} \quad (21)$$

$N_4 = p_{01}p_{13.7} + p_{02}p_{11.8}$ and D_2 is already mentioned.

8. Expected Number of Visits by the Server

Let N_i (t) be the expected number of visits by the server in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for $N_i(t)$ are given as

$$N_i(t) = \sum_j Q_{i,j}^{(n)}(t)(S)\left[\delta_j + N_j(t)\right] \quad (22)$$

where j is any regenerative state to which the given regenerative state *i* transits and δ**j** =1, if *j* is the regenerative state where the server does job afresh, otherwise δ**j** =0.

Taking LST of relation (22) and solving for $\tilde{N}_0(s)$. The expected numbers of visits per unit time by the server are given by

$$N_0(\infty) = \lim_{s\to 0} s\tilde{N}_0(s) = \frac{N_5}{D_2}, \text{ where} \quad (23)$$

$N_5 = (1- p_{11.8})(1-p_{33.10})-p_{13.7}\, p_{3,10}$ and D_2 is already specified.

9. Profit Analysis

The profit incurred to the system model in steady state can be obtained as

$$P = K_0A_0 - K_1B_0^H - K_2B_0^S - K_3R_0 - K_4N_0 \quad (24)$$

where

K_0 = Revenue per unit up-time of the system

K_1 = Cost per unit time for which server is busy due to hardware repair

K_2 = Cost per unit time for which server is busy due to software replacement

K_3 = Cost per unit software replacement

K_4 = Cost per unit visit by the server and $A_0, B_0^H, B_0^S, R_0, N_0$ are already defined.

10. Conclusion

For the particular case g(t) = $\alpha e^{-\alpha t}$, $f(t) = \theta e^{-\theta t}$ and $w(t) = \beta e^{-\beta t}$, it is found that MTSF, availability and profit of the system model go on decreasing with the increase of h/w and s/w failure rates (λ_1 and λ_2) for fixed values of other parameters including a=.7 and b=.3 as shown respectively in figures 2, 3 and 4. However, their values increase with the increase of h/w repair rate (α), s/w replacement rate (Θ) and arrival rate of the server (β).

Thus, it is concluded that profit of a computer system in which chances of h/w failure are higher can be improved up to a considerable level by increasing the arrival rate of the server at s/w failure.

11. References

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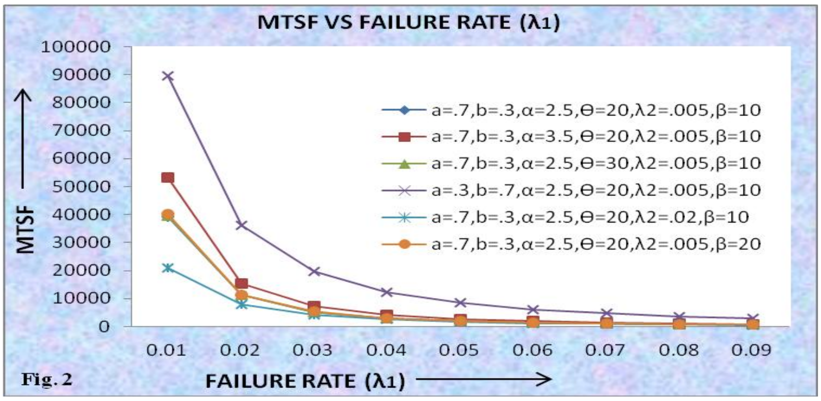

Fig. 2

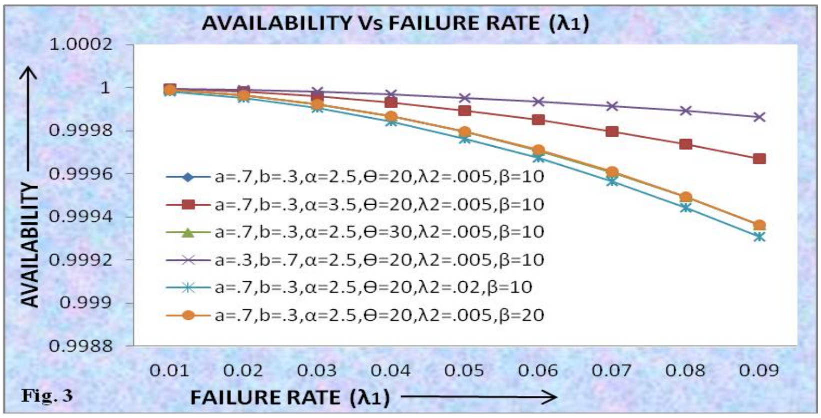

Fig. 3

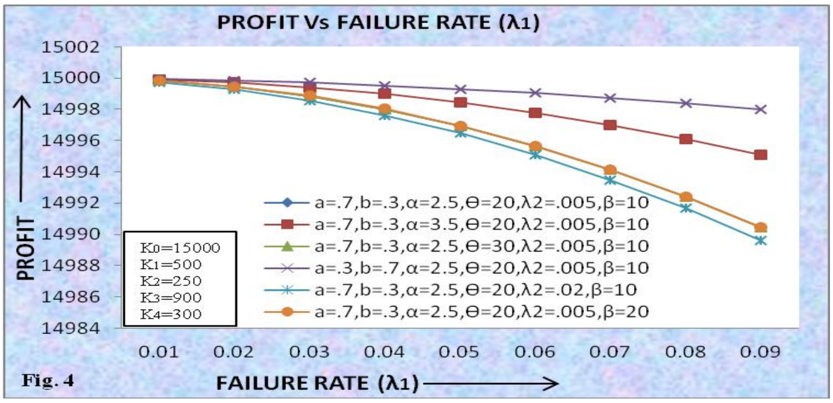

Fig. 4