Analysis of a Computer System with Arrival Time of the Server and Priority to H/w Repair over S/w Replacement

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Abstract

The purpose of this paper is to analyse a computer system considering the aspects of redundancy, priority in repair disciplines and arrival time of the server. Two identical units of a computer system are taken up in which one unit is initially operative and the other is kept as spare in cold standby. In each unit h/w and s/w work together and fail independently from normal mode. There is a single server who takes some time to arrive at the system for doing repair and replacement of the components. Server repairs the unit at its h/w failure while replacement of the s/w is made by new one giving some replacement time in case s/w fails to execute the programmes properly. Priority to the h/w repair is given over the s/w replacement. All random variables are uncorrelated to each other. Repair and switch devices are perfect. The time to failure of the unit due to h/w and s/w is exponentially distributed while the distributions of repair, replacement and arrival times of the server are taken as arbitrary with different probability density functions (pdf). Some reliability characteristics of the system model are derived in steady state using semi-Markov process and regenerative point technique. The numerical results for MTSF, availability and profit are obtained considering a particular case to know their behaviour with respect to different parameters.

Keywords: Computer System, H/w Repair, S/w Replacement, Arrival Time of Server, Priority and Profit Analysis.
Subject Classification: Primary 90B25 and Secondary 60K110.

1. Introduction

Now a day's computer systems are being used at large scale in most of the industrial and management sectors to achieve various complex and safety critical missions. The applications of these systems have also now crossed many other important fields such as air traffic control, nuclear reactors, aircraft, automotive mechanical and safety control, telephone switching and hospital patient monitoring systems. With the development of computer technology, the size and complexity of the computer systems keep on increasing from one single processor to multiple distributed processors, from individual systems to networked systems, from small-scale program running to large-scale resource sharing and from local-area computation to global-area collaboration. Thus an overall assessment of the reliability of computer systems is necessary to provide better services to the customers.

Various techniques have been suggested by the researchers to improve the reliability of such complex systems. The method of redundancy has been proved as one of the best technique in improving the reliability and performance of the operating systems. However, a little work on reliability modelling of computer systems with redundancy has been done by the researchers including Malik and Anand [2010, 11, and 2012] and Malik and Sureria [2012]. These models have been discussed under the assumption that server can be made available immediately as and when required. But, practically, this assumption seems to be unrealistic because it would be very difficult for a server to reach at the system immediately when he is already busy in completion of pre-assigned jobs. And, in such situations, server may take some time to arrive at the system.

In view of above, the present paper deals with the analysis of a computer system considering the concepts of redundancy, priority in repair disciplines and arrival time of the server. Two identical units of the computer system are taken up in which one unit is initially operative and the other is kept as spare in cold standby. In each unit h/w and s/w work together and fails independently from normal mode. There is a single server who takes some time to arrive at the system for doing repair and replacement of the components. Server repairs the unit at its h/w failure while replacement of the s/w is made by new one by giving some replacement time in case s/w fails to execute the programmes properly. Priority to the h/w repair is given over the s/w replacement. All random variables are uncorrelated to each other. Repair and switch devices are perfect. The time to failure of the unit due to h/w and s/w is exponentially distributed while the distributions of repair, replacement and arrival times of the server are taken as arbitrary with different probability density functions (pdf).To analyze the system economically in detail, expression for some reliability characteristics such as mean sojourn times, mean time to system failure (MTSF), availability, busy period of the server due to h/w repair or due to s/w replacement, expected number of replacements due to s/w replacement and expected number of visits by the server are derived by making use of semi-Markov process and regenerative point technique. The graphs are drawn for a particular case to show the behaviour of MTSF, availability and profit of the system models.

2. Notations

E	:	The set of regenerative states
O	:	The unit is operative and in normal mode
Cs	:	The unit is cold standby
a/b	:	Probability that the system has hardware / software failure
λ_1/λ_2	:	Constant hardware / software failure rate
FHUr/FHUR	:	The unit is failed due to hardware and is under repair / under repair continuously from previous state
FHWr / FHWR	:	The unit is failed due to hardware and is waiting for repair/waiting for repair continuously from previous state

FSURp/FSURP	:	The unit is failed due to the software and is under replacement/under replacement continuously from previous state
FSWRp/FSWRP	:	The unit is failed due to the software and is waiting for replacement / waiting for replacement continuously from previous state
w(t) / W(t)	:	pdf / cdf of waiting time of unit due to h/w and s/w failure
f(t) / F(t)	:	pdf / cdf of replacement time of the software
g(t) / G(t)	:	pdf / cdf of repair time of the unit due to hardware failure
q_{ij} (t)/ Q_{ij}(t)	:	pdf / cdf of passage time from regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in (0, t]
$q_{ij.kr}$ (t)/$Q_{ij.kr}$(t)	:	pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k, r once in (0, t]
m_{ij}	:	Contribution to mean sojourn time (μ_i) in state S_i when system transits directly to state S_j so that $\mu_i = \sum_j m_{ij}$ and $m_{ij} = \int t dQ_{ij}(t) = -q_{ij}^{*}{}'(0)$
Ⓢ/©	:	Symbol for Laplace-Stieltjes convolution/Laplace convolution
~ / *	:	Symbol for Laplace Steiltjes Transform (LST) / Laplace Transform (LT)
' (desh)	:	Used to represent alternative result

The following are the possible transition states of the system:
S_0= (O, Cs), S_1= (O, FHWr), S_2= (O, FSWRp), S_3= (O, FSURp), S_4= (O, FHUr), S_5= (FHWr, FHWR), S_6= (FHWR, FHUr),S_7 = (FHWr, FHUR), S_8 = (FSWRp, FHWR), S_9= (FSWRP, FHUr), S_{10}= (FSWRp, FHUR), S_{11}= (FHUr, FSWRP), S_{12}= (FHWr, FSWRP), S_{13}= (FSWRp, FSWRP), S_{14}= (FSWRP, FSURp), S_{15}= (FHUr, FSWRp), S_{16}= (FSWRp, FSURP),
The states S_0–S_4 is regenerative states while the states S_5–S_{16} are non-regenerative as shown in figure 1.

State Transition Diagram

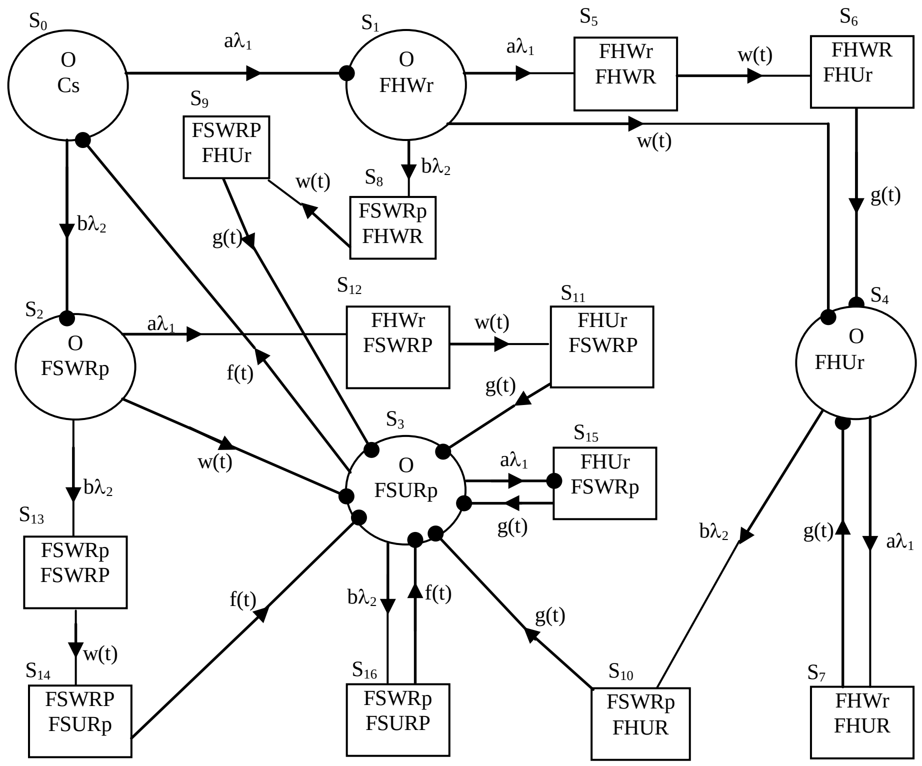

Fig. 1

○ Up-state □ Failed state ● Regenerative point

3. Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements:

$$p_{ij} = Q_{ij}(\infty) = \int_0^{\infty} q_{ij}(t)dt \quad \text{as}$$

$p_{01}=\frac{a\lambda_1}{a\lambda_1+b\lambda_2}$, $p_{02}=\frac{b\lambda_2}{a\lambda_1+b\lambda_2}$, $p_{14}=w^*(a\lambda_1+b\lambda_2)$, $p_{18}=\frac{b\lambda_2}{a\lambda_1+b\lambda_2}\left[1-w^*(a\lambda_1+b\lambda_2)\right]$,

$p_{15}=\frac{a\lambda_1}{a\lambda_1+b\lambda_2}\left[1-w^*(a\lambda_1+b\lambda_2)\right]$, $p_{23}=w^*(a\lambda_1+b\lambda_2)$, $p_{2,13}=\frac{b\lambda_2}{a\lambda_1+b\lambda_2}\left[1-w^*(a\lambda_1+b\lambda_2)\right]$,

$p_{2,12}=\frac{a\lambda_1}{a\lambda_1+b\lambda_2}\left[1-w^*(a\lambda_1+b\lambda_2)\right]$, $p_{30}=f^*(a\lambda_1+b\lambda_2)$, $p_{3,15}=\frac{a\lambda_1}{a\lambda_1+b\lambda_2}\left[1-f^*(a\lambda_1+b\lambda_2)\right]$,

$p_{3,16}=\frac{b\lambda_2}{a\lambda_1+b\lambda_2}\left[1-f^*(a\lambda_1+b\lambda_2)\right]$, $p_{40}=g^*(a\lambda_1+b\lambda_2)$, $p_{47}=\frac{a\lambda_1}{a\lambda_1+b\lambda_2}\left[1-g^*(a\lambda_1+b\lambda_2)\right]$,

$p_{4,10}=\frac{b\lambda_2}{a\lambda_1+b\lambda_2}\left[1-g^*(a\lambda_1+b\lambda_2)\right]$, $p_{56}=w^*(s)$, $p_{64}=g^*(s)$, $p_{74}=g^*(s)$,

$p_{89}=w^*(s)$, $p_{93}=g^*(s)$, $p_{10,3}=g^*(s)$, $p_{11,4}=f^*(s)$, $p_{12,11}=w^*(s)$, $p_{13,14}=w^*(s)$,

$p_{14,3}=f^*(s)$, $p_{15,4}=f^*(s)$, $p_{16,3}=f^*(s)$, $p_{15,3}=g^*(s)$ (1)

For f (t) $=\theta e^{-\theta t}$, g (t) = $\alpha e^{-\alpha t}$ and $w(t)=\beta e^{-\beta t}$ we have

$p_{13.89}=\frac{b\lambda_2}{a\lambda_1+b\lambda_2+\beta}$, $p_{14.56}=\frac{a\lambda_1}{a\lambda_1+b\lambda_2+\beta}$, $p_{23.13,14}=\frac{b\lambda_2}{a\lambda_1+b\lambda_2+\beta}$, $p_{24.12,11}=\frac{a\lambda_1}{a\lambda_1+b\lambda_2+\beta}$,

$p_{33.16}=\frac{b\lambda_2}{a\lambda_1+b\lambda_2+\theta}$, $p_{34.15}=\frac{a\lambda_1}{a\lambda_1+b\lambda_2+\theta}$, $p_{44.7}=\frac{a\lambda_1}{a\lambda_1+b\lambda_2+\alpha}$, $p_{43.10}=\frac{b\lambda_2}{a\lambda_1+b\lambda_2+\alpha}$ (2)

It can be easily verified that $p_{01}+p_{02}=p_{14}+p_{15}+p_{18}=p_{23}+p_{2,12}+p_{2,13}=p_{30}+p_{3,15}+p_{3,16}=p_{40}+p_{47}+p_{4,10}=p_{14}+p_{14.56}+p_{13.89}=p_{23}+p_{23.13,14}+p_{24.12,11}=p_{30}+p_{33.16}+p_{34.15}=p_{40}+p_{44.7}+p_{43.10}=1$ (3)

The mean sojourn times (μ_i) in the state S_i are

$\mu_0=\frac{1}{a\lambda_1+b\lambda_2}$, $\mu_1=\mu_2=\frac{1}{a\lambda_1+b\lambda_2+\beta}$, $\mu_3=\frac{1}{a\lambda_1+b\lambda_2+\theta}$, $\mu_4=\frac{1}{a\lambda_1+b\lambda_2+\alpha}$ (4)

also

$m_{01}+m_{02}=\mu_0$, $m_{14}+m_{15}+m_{18}=\mu_1$, $m_{23}+m_{2,12}+m_{2,13}=\mu_2$, $m_{30}+m_{3,15}+m_{3,16}=\mu_3$, $m_{40}+m_{47}+m_{4,10}=\mu_4$ (5)

$m_{14}+m_{14.56}+m_{13.89}=\mu_1'$, $m_{23}+m_{23.13,14}+m_{23.12,11}=\mu_2'$, $m_{30}+m_{33.16}+m_{3,15}=\mu_3'$, $m_{40}+m_{44.7}+m_{43.10}=\mu_4'$ (6)

for f (t) $=\theta e^{-\theta t}$, g(t)= $\alpha e^{-\alpha t}$ and $w(t)=\beta e^{-\beta t}$

we have $\mu_1'=\frac{\alpha(\alpha+a\lambda_1+b\lambda_2)(\beta+1)+\beta(a\lambda_1+b\lambda_2)(\beta+b\lambda_2+a\lambda_1)}{\alpha\beta(a\lambda_1+b\lambda_2+\alpha)(\beta+b\lambda_2+a\lambda_1)}$,

$\mu_2'=\frac{\theta(\beta+a\lambda_1+b\lambda_2)+\beta(a\lambda_1+b\lambda_2)}{\theta\beta(\beta+b\lambda_2+a\lambda_1)}$ $\mu_3^1=\frac{\theta+b\lambda_2}{\theta(\theta+a\lambda_1+b\lambda_2)}$, $\mu_4^1=\mu_{15}=\frac{1}{\alpha}$ (7)

4. Reliability and Mean Time to System Failure (MTSF)

Let ϕ_i(t) be the c.d.f. of first passage time from regenerative state i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for ϕ_i(t):

$$\phi_i(t)=\sum_j Q_{i,j}(t)(S)\phi_j(t)+\sum_k Q_{i,k}(t) \quad (8)$$

where *j* is an un-failed regenerative state to which the given regenerative state *i* can transit and *k* is a failed state to which the state *i* can transit directly. Taking LST of above relation (8) and solving for $\tilde{\phi}_0(s)$

We have

$$R^*(s)=\frac{1-\tilde{\phi}_0(s)}{s} \quad (9)$$

The reliability of the system model can be obtained by taking Laplace inverse transform of (9). The mean time to system failure (MTSF) is given by

$$\text{MTSF}=\lim_{s\to o}\frac{1-\tilde{\phi}_0(s)}{s}=\frac{N_1}{D_1}, \text{ where} \quad (10)$$

$N_1=\mu_0+p_{01}\mu_1+p_{02}\mu_2+p_{02}p_{23}\mu_3+p_{01}p_{14}\mu_4$ and $D_1=1-p_{01}p_{14}p_{40}-p_{02}p_{23}p_{30}$

5. Steady State Availability

Let A_i (t) be the probability that the system is in up-state at instant't' given that the system entered regenerative state i at t = 0. The recursive relations for A_i (t) are given as

$$A_i(t) = M_i(t) + \sum_j q_{i,j}^{(n)}(t) © A_j(t) \tag{11}$$

where *j* is any successive regenerative state to which the regenerative state *i* can transit through n transitions and $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state, we have

$$M_0(t) = e^{-(a\lambda_1+b\lambda_2)t},\ M_1(t) = M_2(t) = e^{-(a\lambda_1+b\lambda_2)t}\overline{W}(t),\ M_3(t) = e^{-(a\lambda_1+b\lambda_2)t}\overline{F}(t),\ M_4(t) = e^{-(a\lambda_1+b\lambda_2)t}\overline{G}(t) \tag{12}$$

Taking LT of above relations (11) and solving for $A_0^*(s)$, the steady state availability is given by

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2},\ \text{where} \tag{13}$$

$N_2 = p_{30}(1-p_{44.7})[\mu_0 + p_{01}\mu_1 + p_{02}\mu_2] + [1-p_{47.7}-p_{01}p_{40}(1-p_{13.89})]\mu_3 + p_{01}p_{30}(1-p_{13.89})\mu_4$

$D_2 = p_{30}(1-p_{44.7})[\mu_0 + p_{01}\mu_1' + p_{02}\mu_2'] + [1-p_{47.7}-p_{01}p_{40}(1-p_{13.89})][\mu_3' + \mu_{15}'] + p_{01}p_{30}(1-p_{13.89})\mu_4'$

6. Busy Period Analysis for Server

(a) Due to Hardware Repair

Let $B_i^H(t)$ be the probability that the server is busy in repairing the unit due to hardware failure at an instant 't' given that the system entered state i at t = 0. The recursive relations $B_i^H(t)$ for are as follows:

$$B_i^H(t) = W_i^H(t) + \sum_j q_{i,j}^{(n)}(t) © B_j^H(t) \tag{14}$$

where *j* is any successive regenerative state to which the regenerative state *i* can transit through n transitions and $W_i^H(t)$ be the probability that the server is busy in state S_i due to hardware failure up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_4^H(t) = e^{-(a\lambda_1+b\lambda_2)t}\overline{G}(t) + \left[a\lambda_1 e^{-(a\lambda_1+b\lambda_2)} © 1\right]\overline{G}(t) + \left[b\lambda_2 e^{-(a\lambda_1+b\lambda_2)t} © 1\right]\overline{G}(t),\ W_{15}^H(t) = \overline{G}(t) \tag{15}$$

(b) Due to Software Replacement

Let B_i^S (t)be the probability that the server is busy due to replacement of the software at an instant 't' given that the system entered the regenerative state i at t = 0. We have the following recursive relations for B_i^S (t):

$$B_i^S(t) = W_i^S(t) + \sum_j q_{i,j}^{(n)}(t) © B_j^S(t) \tag{16}$$

where *j* is any successive regenerative state to which the regenerative state *i* can transit through n transitions and W_i^S (t) be the probability that the server is busy in state S_i due to replacement of the software up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_3^S(t) = e^{-(a\lambda_1+b\lambda_2)t}\overline{F}(t) + (a\lambda_1 e^{-(a\lambda_1+b\lambda_2)t} © 1)\overline{F}(t) + (b\lambda_2 e^{-(a\lambda_1+b\lambda_2)t} © 1)\overline{F}(t) \tag{17}$$

Taking LT of above relations (14) and (16) and solving for $B_0^{*^H}$ (s) and $B_0^{*^S}$ (s), the time for which server is busy due to repair and replacements respectively is given by

$$B_0^H = \lim_{s\to 0} sB_0^{*H}(s) = \frac{N_3^H}{D_2} \tag{18}$$

$$B_0^S = \lim_{s\to 0} sB_0^{*S}(s) = \frac{N_3^S}{D_2},\ \text{where} \tag{19}$$

$$N_3^H = p_{01}p_{30}(1-p_{13.89})\tilde{W}_4^H(0) + p_{3,15}[p_{43.10} + p_{02}p_{04} + p_{01}p_{40}p_{13.89}]\tilde{W}_{15}^H(0)$$

$N_3^S = [p_{43.10} + p_{02}p_{04} + p_{01}p_{40}p_{13.89}]\tilde{W}_3^S(0)$ and D_2 is already mentioned.

7. Expected Number of Software Replacements

Let $R_i^S(t)$ be the expected number of replacements of the failed software by the server in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for $R_i^S(t)$ are given as

$$R_i^S(t) = \sum_j Q_{i,j}^{(n)}(t)(S)\left[\delta_j + R_j^S(t)\right] \tag{20}$$

Where j is any regenerative state to which the given regenerative state *i* transits and $\delta j=1$, if *j* is the regenerative state where the server does job afresh, otherwise $\delta j= 0$. Taking LST of relations (20) and solving for $\tilde{R}_0^S(s)$. The expected number of replacements per unit time to the software failures is given by

$$R_0(\infty) = \lim_{s\to 0} s\tilde{R}_0^S(s) = \frac{N_4}{D_2}, \text{ where} \quad (21)$$

$N_4 = (1 - p_{13.89})[p_{43.10} + p_{02}p_{04} + p_{01}p_{40}p_{13.89}]$ and D_2 is already mentioned.

8. Expected Number of Visits by the Server

Let N_i (t) be the expected number of visits by the server in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for $N_i(t)$ are given as

$$N_i\ (t) = \sum_j Q_{i,j}^{(n)}(t)(S)\left[\delta_j + N_j(t)\right] \quad (22)$$

where j is any regenerative state to which the given regenerative state *i* transits and δj =1, if *j* is the regenerative state where the server does job afresh, otherwise δj = 0. Taking LST of relation (22) and solving for $\tilde{N}_0(s)$. The expected numbers of visits per unit time by the server are given by

$$N_0(\infty) = \lim_{s\to 0} s\tilde{N}_0(s) = \frac{N_5}{D_2}, \text{ where} \quad (23)$$

$N_5 = (1- p_{44.7})\, p_{30}$ and D_2 is already specified.

9. Cost-Benefit Analysis

The profit incurred to the system model in steady state can be obtained as

$$P = K_0A_0 - K_1B_0^H - K_2B_0^S - K_3R_0 - K_4N_0 \quad (24)$$

where

K_0 = Revenue per unit up-time of the system

K_1 = Cost per unit time for which server is busy due to hardware repair

K_2 = Cost per unit time for which server is busy due to software replacement

K_3 = Cost per unit software replacement

K_4 = Cost per unit visit by the server and $A_0, B_0^H, B_0^S, R_0, N_0$ are already defined.

10. Conclusion

By assuming g (t) $=\alpha e^{-\alpha t}$, $f(t) = \theta e^{-\theta t}$ and $w(t) = \beta e^{-\beta t}$, the results for some performance measures of a computer system are obtained giving particular values to various parameters and costs. It is observed that mean time to system failure (MTSF), availability and profit go on decreasing with the increase of h/w and s/w failure rates (λ_1 and λ_2) as shown in figures 2, 3 and 4 respectively. However, the values of these measures increase with the increase of repair rate (α) and arrival rate (β) of the server.

Hence, it is concluded that reliability and profit of a computer system can be improved by the technique of redundancy and by giving priority to h/w repair over s/w replacement.

11. References

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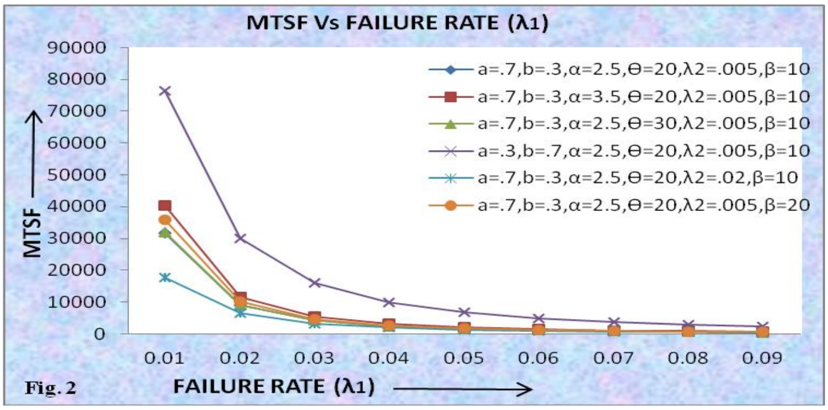

Fig. 2

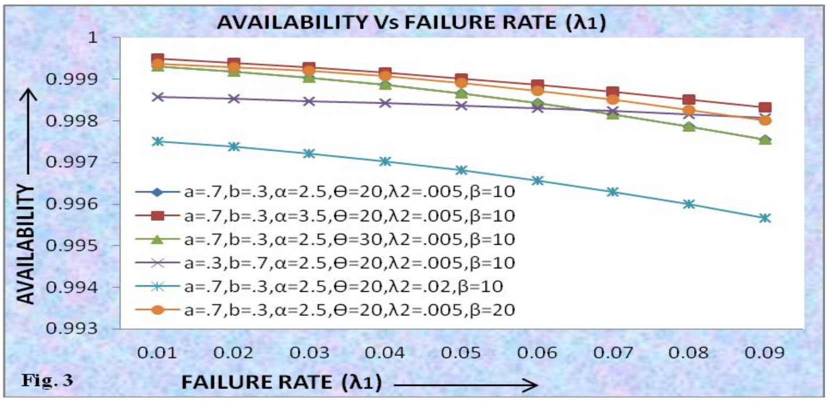

Fig. 3

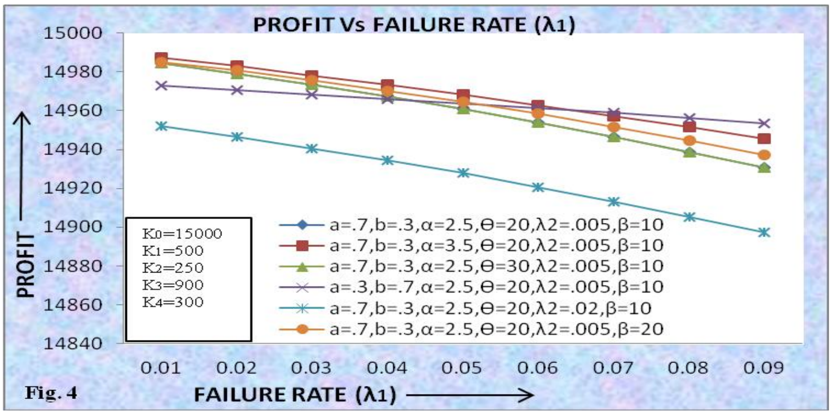

Fig. 4