Economic Efficiency of the LTPD Sampling Plans for Inspection by Variables when the Remainder of Rejected Lots is Inspected

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Abstract

LTPD sampling plans minimizing the mean inspection cost per lot of process average quality when the remainder of rejected lots is inspected were originally designed for the inspection by attributes. The single sampling plans for the inspection by variables and for the inspection by variables and attributes (all items from the sample are inspected by variables, remainder of rejected lots is inspected by attributes) were then proposed. Under the same protection of consumer the LTPD plans for inspection by variables are in many situations more economical than the corresponding sampling plans for inspection by attributes. Comparative efficiency of sampling plan for inspection by variables and attributes can be assessed using a measure which has been proposed for this purpose.

Keywords: acceptance sampling, consumer's risk, cost of inspection, single sampling

1. Introduction

Under the assumption that each inspected item is classified as either good or defective (acceptance sampling by attributes), Dodge and Romig (1998) consider sampling plans which minimize the mean number of items inspected per lot of process average quality

$$I_s = N - (N - n) \cdot L(\bar{p}; n; c) \tag{1}$$

under the condition

$$L(p_t; n; c) \leq \beta \tag{2}$$

where $L(p, n, c)$ is the operating characteristic (the probability of accepting a submitted lot with proportion defective p when using plan (n, c) for acceptance sampling), N is the number of items in the lot (the given parameter), $\bar{p}$ is the process average proportion defective (the given parameter), p_t is the lot tolerance proportion defective (the given parameter, $P_t = 100p_t$ is the lot tolerance per cent defective, denoted LTPD), n is the number of items in the sample $(n < N)$, c is the acceptance number (the lot is rejected when the number of defective items in the sample is greater than c). Condition (2) provides a guarantee for the consumer that lots of unsatisfactory quality level, with proportion defective p_t are going to be accepted only with the specified probability β (consumers risk). Value 0.1 is used for consumer's risk in Dodge and Romig (1998). LTPD plans for the inspection by variables and attributes have been introduced in (Klufa, 1994), using an approximate calculation of the plans. Exact plans, using non-central t distribution in calculation of the operating characteristic, have been reported in (Klufa, 2010). This paper shows economic characteristics of the exact LTPD plans for the inspection by variables and for the inspection by variables and attributes and shows the impact of input parameters values on resulting sampling plan and its economic efficiency. A measure for the assessment of the economic efficiency of these plans is proposed.

2. Calculation and economic characteristics of LTPD plans for the inspection by variables and attributes

We will calculate LTPD acceptance sampling plan for the sampling inspection by variables when the remainder of rejected lots is inspected by attributes. The resulting sampling plan will be evaluated with regard to the economic characteristics and compared with the corresponding Dodge-Romig plan in (Dodge and Romig, 1998).

Example. A lot with $N = 1000$ items is considered in the acceptance procedure.The lot tolerance proportion defective is given to be $p_t = 0.01$ and the consumer's risk $\beta = 0.1$. It is known that the average process quality is $\bar{p} = 0.001$. A cost of inspecting an item by variables is 50% higher than the cost of inspecting the item by attributes, so the parameter c_m equals 1.5. Find LTPD acceptance sampling plan for sampling inspection by variables when the remainder of rejected lots is inspected by attributes.

The LTPDvar package (Kasprikova, 2012) for R software (R Development Core Team, 2013) can be used for the calculation of the plan. The resulting plan is $n = 77$, $k = 2.644489$. For the values of the input parameters given in our problem, there is plan (205, 0) for the acceptance sampling by attributes in Dodge and Romig (1998). Let us compare the plans ($n = 77$, $k = 2.644489$) and ($n = 205$, $c = 0$) with regard to the economic efficiency.

For the comparison of the LTPD sampling plans for the inspection by variables and attributes and the corresponding Dodge-Romig LTPD sampling plans for the inspection by attributes with regard to the economic point of view we will use parameter e, defined as

$$e = \frac{I_{ms}}{I_m} \cdot 100 \tag{3}$$

where $I_{ms} = n \cdot c_m + (N - n) \cdot (1 - L(\bar{p}; n; k))$.

The expression $(1 - e)$ then represents the percentage of savings in the inspection cost when the sampling plan for the inspection by variables and attributes is used instead of the corresponding plan for the inspection by attributes. Let us denote the plan for the inspection by variables and attributes as (n_1, k) and the corresponding plan for the inspection by attributes as $(n_2,\ c)$ then it is

$$e = \frac{n_1 \cdot c_m + (N - n_1) \cdot (1 - L(\bar{p}; n_1; k))}{N - (N - n_2) \cdot L(\bar{p}; n_2; c)} \cdot 100. \tag{4}$$

Since for $(n_1,\ k) = (77,\ 2.644489)$ and $(n_2,\ c) = (205{,}0)$, we get $e = 39.4$, it can be expected that approximately 60% savings in theinspection cost can be achieved using the LTPD plan for the inspection by variables and attributes in place of the corresponding Dodge-Romig plan. Economical parameter e defined by (3) is a function of four variables p_t, N, $\bar{p}$ and c_m, i. e. $e = e(p_t, N, \bar{p}, c_m)$.

Let p_t, N, $\bar{p}$ be given parameters (for given p_t, N, $\bar{p}$ we shall write $e_{p_t,N,\bar{p}}(c_m)$ instead of $e = e(p_t, N, \bar{p}, c_m)$). Function $e_{p_t,N,\bar{p}}(c_m)$ (i.e. e for given parameters p_t, N, $\bar{p}$) is a function of one variable c_m.

Remark. From the results of numerical investigations it follows that the function $e(p_t, N, \bar{p})$ (i.e. e for given parameters p_t, N, $\bar{p}$) is increasing function of c_m (see also Figure 1).

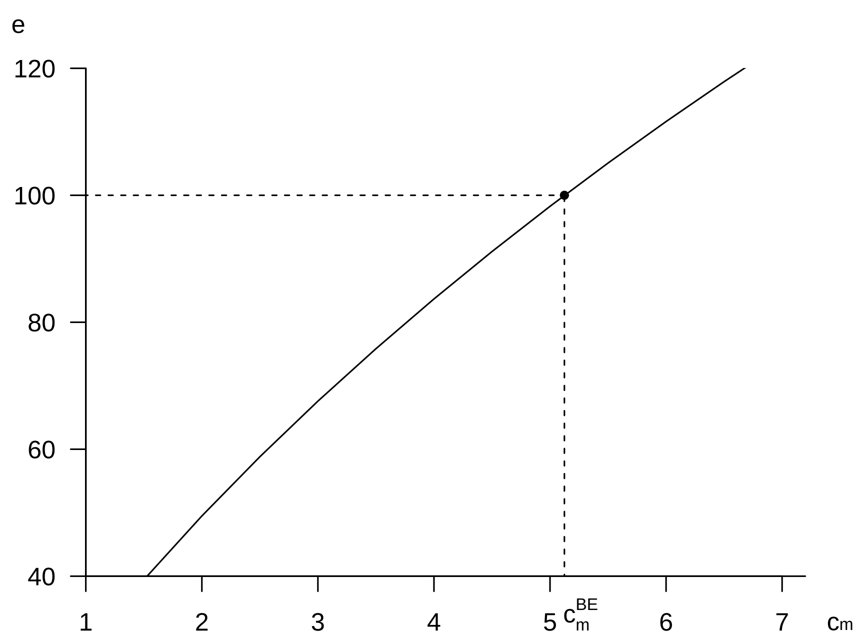

Figure 1: Graph of function $e = e(c_m)$ for p_t =0.01, N =1000, $\bar{p}$ =0.001

Definition. Let p_t, N, $\bar{p}$ be given parameters . Let us define c_m^{BE} as the value of c_m for which $e = 100$.

According to this definition, c_m^{BE} is such value of c_m for which the mean inspection cost per lot of process average quality for the inspection by variables and attributes is equal to the mean inspection cost per lot of process average quality for the inspection by attributes. If c_m is an estimate based on real cost calculation and $c_m < c_m^{BE}$, then the LTPD plans for inspection by variables and attributes are more economical than the corresponding Dodge-Romig LTPD plans. If $c_m > c_m^{BE}$, then the Dodge-Romig LTPD plans for inspection by attributes are more economical than the corresponding LTPD plans for the inspection by variables and attributes.

If the value of c_m parameter is not known in some situation in practice, then c_m^{BE} (the break-even value of c_m parameter) may be calculated to provide some guidance in deciding if inspection by variables and attributes is worth considering. If c_m^{BE} is high, then using inspection by variables and attributes may be efficient (and one should try to estimate c_m to make some more precise evaluation), on the other hand if c_m^{BE} is near 1, then the inspection by variables and attributes can not be supposed to bring significant advantage over the inspection by attributes. The calculation of c_m^{BE} value is implemented in the LTPDvar package.

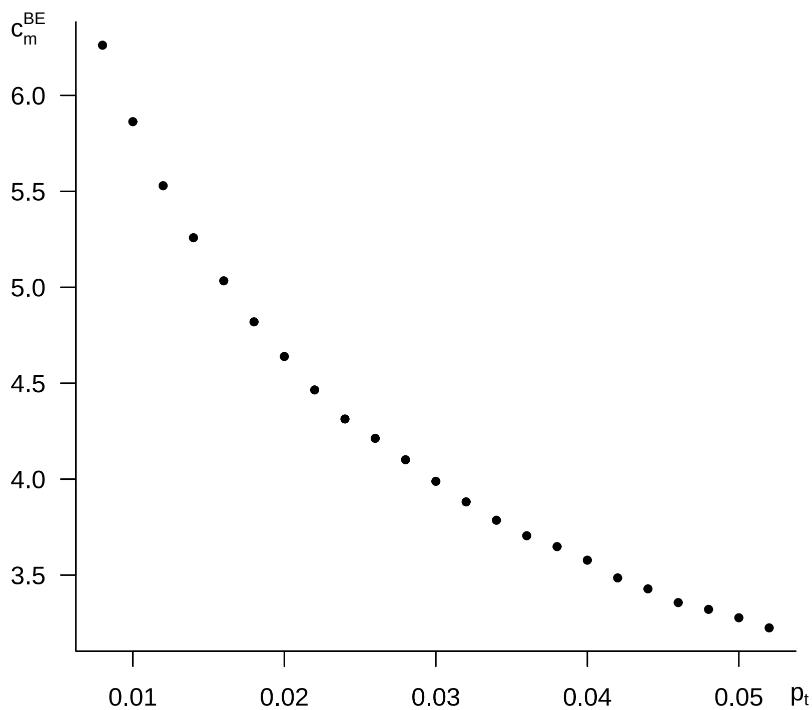

Figure 2: Graph of function $c_m^{BE} = c_m^{BE}(p_t)$ for N =2000, $\bar{p}$ =0.0005

Example. For the values of the input parameters p_t=0.01, N=1000, $\bar{p}$=0.001, considered in our example, the resulting c_m^{BE} value is

$$c_m^{BE} = 5.126.$$

The parameter c_m^{BE} is a function of three variables p_t, N, $\bar{p}$. From the results of numerical investigations it seems that (see also Figure 2) it has increasing trend in N and decreasing trend in p_t. It means that the economic efficiency of the LTPD plans for inspection by variables and attributes roughly speaking increases when the lot size N is increasing and decreases when the lot tolerance proportion defective p_t increases.

3. Conclusions

LTPD acceptance sampling plans for the inspection by variables and for the inspection by variables and attributes may bring substantial savings in the inspection cost in comparison with the corresponding Dodge-Romig acceptance sampling plans for the inspection by attributes and the extent of savings in the inspection cost is given by the input parameters values such as the ratio of the cost of inspection of one item by variables to the cost of inspection of the item by attributes c_m. Smaller values of this ratio lead to higher savings in inspection cost. In case of unknown c_m parameter, the economic efficiency of the sampling plan for the inspection by variables and attributes in comparison with the plan for the inspection by attributes may be assessed using the break even value of c_m parameter.

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