Extensions of the Autoregressive Sieve Bootstrap to Multivariate and Spatial Time Series

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Extending the results of Kreiss, Paparoditis and Politis (2011), we explore the limits of the autoregressive (AR) sieve bootstrap. This procedure fits a finite-order AR model to the given data and uses this fit to generate bootstrap replicates of the time series. Based on these bootstrap samples the distribution of many statistics like the sample mean or sample autocorrelations can be approximated. The order of the fitted AR model depends on the number of given observations and increases as the sample size tends to infinity. For certain statistics and data stemming from a large class of processes, the validity of the procedure can be shown. This is due to the fact that, under relatively mild conditions, many processes inherit a certain autoregressive structure which is mimicked correctly by the AR sieve bootstrap. With this work, we extend these results to the case of multivariate time series and spatial data (random fields). The goal is to provide a general check criterion for the validity of the bootstrap which enables us to decide whether the procedure does or does not work for a specific statistic. In the latter case it should also point out the exact reason which causes the bootstrap to fail. This can be used as a starting point to implement specific modifications to the procedure in order to compensate its shortcomings.

Key Words: Time series analysis, autoregressive representations, random fields