Delta Method on Bootstrapping of Autoregressive Process

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Let $\{X_t, t \in T\}$ be autoregressive time series, where T is discrete time, and let $X_1, X_2, \ldots, X_n$ be the sample that satisfies the AR(1) process. Thus, the sample follows the relation $X_t = \theta X_{t-1} + \varepsilon_t$ where $\{\varepsilon_t\}$ is a zero mean white noise process with constant variance σ^2. Let $\hat{\theta}$ be the estimator for parameter θ. Brockwell and Davis (1991) showed that $\hat{\theta} \to_p \theta$ and $\sqrt{n}(\hat{\theta} - \theta) \to_d N(0, \sigma^2)$. Meantime, by some assumptions, can be showed that the distribution of $\sqrt{n}(\bar{X} - \mu)$ converges to normal distribution with mean *0* and variance σ^2 as $n \to \infty$. In bootstrap view, the key of bootstrap terminology says that the population is to the sample as the sample is to the bootstrap samples. Therefore, when we want to investigate the consistency of the interesting bootstrap estimator for sample mean, we investigate the distribution of $\sqrt{n}(\bar{X}^* - \bar{X})$ contrast to $\sqrt{n}(\bar{X} - \mu)$, where $\bar{X}^*$ is bootstrap version of $\bar{X}$ computed from sample bootstrap X^*. Asymptotic theory of the bootstrap sample mean is useful to study the consistency for many other statistics. Let $\hat{\theta}^*$ be the bootstrap estimator for $\hat{\theta}$. In this paper we investigate the consistency of $\hat{\theta}^*$ using delta method and applying the residuals bootstrap. We also present the Monte Carlo simulations in regard to yield apparent conclusions.

Key Words*:* Bootstrap, consistency, time series, delta method, Monte Carlo simulations