Holonomic Descent Minimization Method for the Restricted Maximum Likelihood Estimation

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Recently, the school of Takemura and Takayama have developed a quite interesting minimization method called ”holonomic descent minimization method”. It works by a mixed use of Pfaffian differential equation satisfied by an objective holonomic function and iterative Newton optimization method. They successfully applied the method to several maximum likelihood estimation (MLE) problems, which have been intractable in the past. For example, the MLEs are now solvable numerically by the method for the von Mises-Fisher distribution and the Fisher-Bingham distribution on the sphere. Furthermore, the method has also been applied to the evaluation of the exact distribution function of the largest root of a Wishart matrix and is rapidly expanding the area of applications. On the other hand, in statistical models, it is not rare that parameters are restricted and therefore the restricted MLE has been surely one of fundamental topics in statistics. In this paper we develop the holonomic descent MLE methods for the restricted parameter settings. The key is to separate the iterative process into two processes of (A):the update of new parameter values by Newton method with a penalty function or an extended Lagrangian method and (B):solving a Pfaffian system. The performance of the proposed method studied by simulation is presented.

Key Words: Pfaffian system, von Mises-Fisher distribution, Newton method with a penalty function, extended Lagrangian method