

The human capital investment model based on the stochastic control problem

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Abstracts

The present study on the human capital investment is not considered some stochastic risk, so this paper considers the time of dynamic and uncertainty of cost in the process of human capital investment, and puts the problem of optimal stochastic control into the human capital investment model for the first time. It develops the applications of the stochastic control theory, as well as enriches the study of human capital investment decision. This paper summarizes the human capital and human capital investment in research and development, then introduces the related singular stochastic control model and some research methods, and finally it focuses on the human capital investment decision model based on a class of singular stochastic control problem with stopping time.

Keywords: optimal strategy, singular stochastic control, stopping time

1. The establishment and development of human capital and human capital investment theory

From the concept to the theory of human capital, it costs about two centuries. The famous classical economist, Adam Smith, was the first person, who regarded the labor power as the capital. He thought the abilities which all people in the community learned are the part of the property for both individuals and society, and belong to the social fixed capital. At that time, Adam Smith had introduced the idea that the increasing quantity and improving the quality of workers is one of the conditions of the wealth growth. However, due to the limitations of the times, the human capital theory did not immediately established and developed until American economist Theodore Schultz first systematically elaborated the human capital theory in the speech of "Investment in Human Capital" in 1960. Schultz clearly stated that the human factor is the most important among all the factors affecting economic development, and the economic development depends mainly on improving the quality of people rather than the abundance of natural resources or the amount of capital. He also proposed the human capital is the main reason for promoting the national economic growth in the modern era, and the population quality and knowledge investment determine the future of humanity. Another American Economists, Gary S. Becker, who published the book of "Human Capital" in 1964, contributed to study of the human capital theory. Becker analyzed the human resources microscopically, and expounded a various types of investment for human capital formation and its return.

Of course, there are many other scholars engaging in the human capital research, such as David Ricardo and John Stuart Mill, who inherited and developed the thought of Adam Smith. Ricardo insists that the magnitude of commodity value is decided by the labor time, and labor is the only source of value. Mill suggested the skills, knowledge and ability should be regarded as machines and tools as part of the national wealth, which are important factors for the labor productivity. Freidrich Liszt presented the concepts of "Physical Capital" and "Spiritual Capital", and the former is the accumulation of material wealth; while the later is the achievements of human intelligence. Alfred Marshall also proposed that the knowledge and organization are the important parts of the capital and they are the most powerful productivities. In

1906, Irving Fisher put the concept of human capital into the theoretical framework for economic analysis. Robert Lucas combined Schultz's human capital theory with Romer's idea of technological progress by using individual analysis, and the concept of human capital is specified into "everyone" and "professional human capital".

For the concept of the human capital, we reference to Schultz and Becker's ideas. The former thought that human capital is a type of capital reflected in the laborers, and it is represented by the quantity and quality of workers, that is, the knowledge level, technical level, the ability to work and health status. It is the sum of the value of these aspects. While Becker thought the human capital means not only the talents, knowledge and skills, but also the time, health and lifespan.

Human capital is formed by investment, so research on human capital investment is an important element of human capital theory. Becker pointed the investment in human capital is to increase the human resources to influence the future monetary and material income. In particular, investment in human capital is the initial investment which is used to improve the labor's production capacity for improving the earning capacity in the labor market. Schultz has concentrated investment in human capital on five major categories: " 1) health facilities and services, broadly conceived to include all expenditures that affect the life expectancy, strength and stamina, and the vigor and vitality of people; 2) on-the-job training, including old-style apprenticeship organized by firms; 3) formally organized education at the elementary, secondary, and higher levels; 4) study programs for adults that are not organized by firms, including extension programs notably in agriculture; 5) migration of individuals and families to adjust to changing job opportunities." Based on these, later scholars put forward the specific forms of the investment in human capital, including: levels of formal education, on-the-job training activities, a variety of health investment, the time for parents to take care of child, activities for workers to look for job, as well as migration for workers to move from one region to another.

At present, most of the research on human capital investment take the cost - benefit analysis in human capital as the foundation and premise, and usually adopt the net present value method and the internal rate of return method. In these methods, however, all future expected cash flow are believed to happen in the future, while some unpredictable factors are ignored during the human capital investment process, such as the cost of risk and time value. Therefore, some scholars have proposed the application of real options in human capital investment decision-making. Heckman et al (2006) research the call option in human capital investment; Hogan and Walker (2007) discuss the problem in the education investment; Liu et al (2009) introduce the real of option into the human capital investment, but just consider the investment in education. There are many other research on the human capital investment for individuals or enterprises, such as Lee Han-tong (2006) discusses the individual human capital investment behavior under two cases; Sun Jiang-wei et al (2011) discuss the human capital game among enterprises, between enterprises and employees.

For a more comprehensive and reasonable, we will consider the time variable and random disturbance in the human capital investment model, the former is taken into account the dynamic nature of time, and the later is a combination of a variety of uncertainties. We will research the human capital investment decision model by using the stochastic control theory and methods. To illustrate the problem, we introduce the singular stochastic control problem and the related research methods.

2. The singular stochastic control problem

The stochastic control theory combines the stochastic processes theory and the optimal control theory for researching the stochastic system. It applies the theory of stochastic process, stochastic analysis, optimal control, stochastic differential equation and variational inequalities. The stochastic control theory is applied in many fields,

such as the satellite tracing, the financial decision, the security analysis, the risk management, the signal processing and so on. The problem of optimal stochastic control can be described as following: For a dynamic state, the controller chooses the optimal strategy from the permission strategy and the performance index of control system can reach the best.

With the wide application of the optimal stochastic control theory, a class of singular stochastic control with stopping time has been introduced, which proves a set of methods and theories for the research of investment decision making model. Karatzas (1983) first proposed the problem of limited horizon. Davis and Zervos (1994) extended the model as follow,

Let (Ω, F, P) be a probability space with a filtration $F_t = \sigma(W_s, 0 \leq s \leq t)$ satisfying the usual conditions. $W_t, t \geq 0$ is a standard one-dimensional F_t -Brownian motion. Let B denote the set of F_t -adapted, left-continuous, bounded variation processes, and a process $\xi = \{\xi_t : t \geq 0\} \in B$ will be represented as $\xi_t = \xi_t^+ - \xi_t^-$, where $\xi_t^\pm \in B$ are nondecreasing and the representation is minimal, so that the total variation $\check{\xi}_t = \xi_t^+ + \xi_t^-$. Let Γ is set of F_t -stopping time. For $\forall x \in R$, the discount cost function on control strategy (ξ, τ) is following as

$$J_x(\xi, \tau) = E \left\{ \int_0^\tau e^{-\delta t} \left[\lambda X_t^2 dt + d\check{\xi}_t \right] + e^{-\delta \tau} \alpha X_\tau^2 \right\}. \tag{1}$$

The state process is

$$X_t = x + W_t + \xi_t, t \geq 0, \tag{2}$$

It will get the optimal control $\xi^* \in B$ and optimal stopping time $\tau^* \in \Gamma$, so that $J_x(\xi^*, \tau^*) = \inf_{\xi \in B, \tau \in \Gamma} J_x(\xi, \tau)$.

Since 2006, we have been researching the generalization of above model. At first Yu et al (2006, 2008, 2009) add the drift in the state process like equation (2), and extent the diffusion from 1 to the constant $\sigma > 0$, that is the following equation (3),

$$X_t = x + \mu t + \sigma W_t + \xi_t, t \geq 0. \tag{3}$$

Then Yu et al (2010) take the general extension for both the state process and the cost function. The extended state process become the solution of stochastic differential equation as following,

$$dX_t = \mu(X_t)dt + \sigma(X_t)dW_t + d\xi_t, t \geq 0, X_0 = x > 0. \tag{4}$$

And the discount cost function is following as

$$J_x(\xi, \tau) = E \left\{ \int_0^\tau e^{-\delta t} \left[h(X_t)dt + d\check{\xi}_t \right] + e^{-\delta \tau} g(X_\tau) \right\} \tag{5}$$

After strictly proving, we have shown the existence of optimal control ξ^* and the

optimal stopping time τ^* . We also get the analytic expression of the optimal function.

Yu et al (2012) also introduce such problems into financial investment decision model. According to market fluctuations, the investor can choose the time to begin or

stop the investment. In the investment process, it products profit as well as cost, and the cost incurrs when beginning or stopping the investment happen. So the investor's goal is to select the optimal investment strategy for the minimum cost and maximum income. In our model, the commodity price meets the following equation,

$$X_t = x + \int_0^t \mu X_s ds + \int_0^t \sqrt{2}\sigma X_s dW_s - Z_t, \forall t \geq 0 \quad (6)$$

Let the cost function meets

$$J_x(\xi, \tau) = E \left\{ \int_0^\tau e^{-\delta t} [h(X_t)dt + dZ_t] + e^{-\delta \tau} g(X_\tau) \right\} \quad (7)$$

Finally, we get the optimal investment strategy.

3. Conclusions

Compared with the general commodity production investment, the human capital investment possesses the characteristics of continuity, adjustability, long term and uncertainty. Due to these features, it is necessary to consider the factor of risk and time for researching the problems of investment in human capital. While the stochastic control model can just solve these problems. Therefore, we introduce a class of singular stochastic control model with stopping time in human capital investment decision problem, trying to study more comprehensively and solve the problem of human capital investment.

We propose the model of human capital investment. We consider one's final wealth over the life cycle depends on three aspects: the cost of education and training, labor income and the cost for leisure. So our goal is to find the optimal decision to make the wealth maximization. We will solve the following questions: When to end education for work? How to allocate costs of education and others? When to enter the re-education process? So we define one's final wealth as follow,

$$MaxE \left\{ \int_t^T e^{-\rho(s-t)} u [c(s), l(s)] ds + e^{-\rho(T-t)} B(W(T), T) \right\},$$

where $W(t)$ represents the current financial wealth, and

$dW(t) = [rW(t) + (1 - \lambda(t) - l(t))k(t) - c(t) + W(t)X(t)(\mu - r)]dt + W(t)X(t)\sigma'_s dZ(t)$
 $Z(t)$ is the standard Wiener process; $\lambda(t)$ and $l(t)$ represent the proportion of time devoted to education and leisure respectively; $c(t)$ means the consumption expenditures; $k(t)$ is defined as the maximum current income which the individual can expect on the labor market.

Due to the characteristics of human capital investment, it is better for comprehensive analysis of human capital investment to introduce the real option theory in the financial field and the optimal control problems in stochastic analysis. The conclusions of the optimal strategy will be used to the government or enterprise human resources management and education reform scheme. Our next job is to use the research methods of optimal stochastic control theory to solve the problem of human capital investment.

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