The Best Estimation for High-Dimensional Markowitz Mean-Variance Optimization

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Abstract The traditional (plug-in) return for the Markowitz mean-variance (MV) optimization has been demonstrated to seriously overestimate the theoretical optimal return, especially when the dimension to sample size ratio p/n is large. The newly developed bootstrap-corrected estimator corrects the overestimation, but it incurs the "under-prediction problem," it does not do well on the estimation of the corresponding allocation, and it has bigger risk. To circumvent these limitations and to improve the optimal return estimation further, this paper develops the theory of spectral-corrected estimation. We first establish a theorem to explain why the plug-in return greatly overestimates the theoretical optimal return. We prove that under some situations the plug-in return is $\sqrt{\gamma}$ times bigger than the theoretical optimal return, while under other situations, the plug-in return is bigger than but may not be $\sqrt{\gamma}$ times larger than its theoretic counterpart where $\gamma = \frac{1}{1-y}$ with y being the limit of the ratio p/n.

Thereafter, we develop the spectral-corrected estimation for the Markowitz MV model which performs much better than both the plug-in estimation and the bootstrap-corrected estimation not only in terms of the return but also in terms of the allocation and the risk. We further develop properties for our proposed estimation and conduct a simulation to examine the performance of our proposed estimation. Our simulation shows that our proposed estimation not only overcomes the problem of "over-prediction," but also circumvents the "under-prediction," "allocation estimation," and "risk" problems. Our simulation also shows that our proposed spectral-corrected estimation is stable for different values of sample size n, dimension p, and their ratio p/n. In addition, we relax the normality assumption in our proposed estimation so that our proposed spectral-corrected estimators could be obtained when the returns of the assets being studied could follow any distribution under the condition of the existence of the fourth moments.