The link between unemployment and inflation using Johansen's co-integration approach and vector error correction modelling

Sagaren Pillay*

Statistics South Africa, Pretoria, South Africa sagarenp@statssa.gov.za

Abstract

In this paper bi-annual time series data on unemployment rates (from the Labour Force Survey) are expanded to quarterly rates and linked to quarterly unemployment rates (from the Quarterly Labour Force Survey). The resultant linked series and the consumer price index rate (CPI) series are examined using Johansen's co-integration approach and vector error correction modelling. The study finds that both the series are integrated of order one and are co- integrated. A statistically significant co-integrating relationship is found to exist between the time series of unemployment rates and the CPI rate. Given this significant relationship, the study models this relationship using Vector Error Correction Models (VECM), one with a restriction on the deterministic term and the other with no restriction.

A formal statistical confirmation of the existence of a unique linear and lagged relationship between inflation and unemployment for the period between September 2000 and June 2011 is presented. For the given period, the CPI rate was found to be an unbiased predictor of the unemployment rate. This relationship can be explored further for the development of appropriate forecasting models incorporating other study variables.

Key words: forecasting, lagged, linear, relationship.

Introduction

Kitov (2007), carried out several formal cointegration tests for the relationships between inflation, unemployment and labour force change rate and obtained an overall confidence in the existence of a true linear and lagged link between the variables. According to Kitov (2006), the links between inflation and unemployment demonstrate various and even opposite dependencies. In the USA, this dependence is characterized by a positive influence of inflation on unemployment. Effectively, low inflation in the USA leads low unemployment by three years.

In explaining the relationship between inflation and unemployment, Stephanides (2006) found that although there are periods where there is a clear trade-off between inflation and unemployment there are periods where both inflation and unemployment change in the same direction.

In this study, the two variables (unemployment rate and CPI), being non-stationary I(1), were found to be cointegrated in a statistical sense. This means that their residual time series in the vector error correction model (VECM) representation proves to be stationary. The models for both a VECM model with a restriction on the deterministic term and one with no restriction is presented.

In interpreting the two models one must consider that, modelling cointegrated series is difficult because of the need to model systems of equations in which one has to simultaneously specify the deterministic terms and how they enter, determine the lag length, and ensure a congruent representation (Hendry & Juselius 2001).

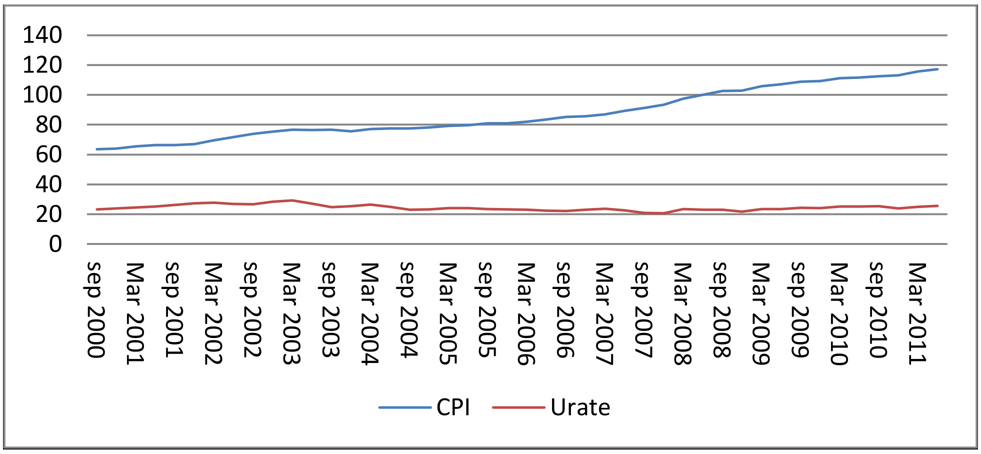
Data

The Labour Force Survey (LFS) was introduced in September 2000 and was published on a biannual basis until March 2008 when the Quarterly Labour Force Survey (QLFS) was introduced. This resulted in a discontinuous series that made analysis of unemployment estimates very difficult.

In this study, the LFS biannual unemployment rate time series from September 2000 to September 2007 and the QLFS quarterly unemployment rate time series for the period March 2008 to June 2011 were combined and adjusted for the purpose of analysis. The biannual unemployment rates were converted to quarterly rates using the SAS procedure PROC EXPAND. The procedure uses the SPLINE method by fitting a cubic spline curve to the input values. A cubic spline is a segmented function consisting of third-degree (cubic) polynomial functions joined together so that the whole curve and its first and second derivatives are continuous (Appendix 1).

The CPI rates (an economic indicator of inflation) were used for matching quarters with the data for the corresponding unemployment rates.

Figure 1: Graphical representation of unemployment rates and CPI rates



The graphical representation (Figure 1) shows the plot of the expanded LFS series combined with the QLFS series (Urate) and the plot of the CPI rates for the corresponding months at the end of each quarter.

Methodology and Results

The first step in the time series analysis was to determine whether the two series are stationary or non-stationary in nature. If the time series are I (1), they have to be characterized by the presence of a unit root and their first difference by the absence of unit roots (Hendry, 2001).

The Augmented Dickey Fuller (ADF) unit root test was used to determine whether the series was stationary or non-stationary. The Dickey- Fuller tests for non stationarity of each of the series is shown below (Table 1). The null hypothesis is to test a unit root. Consequently, both series have a unit root and their first differences do not have any. Thus, the variables URate and CPI are first order difference stationary and are integrated of order I (1) (Table 1).

Table 1

Dickey-Fuller Unit Root Tests					
Variable	**Type**	**Rho**	**Pr < Rho**	**Tau**	**Pr < Tau**
URate	Zero Mean	0.02	0.6828	0.07	0.7002
	Single Mean	-9.54	0.1271	-2.08	0.2553
	Trend	-12.61	0.2292	-2.29	0.4286
CPI	Zero Mean	0.61	0.8260	3.99	0.9999
	Single Mean	0.52	0.9747	0.92	0.9948
	Trend	-2.88	0.9361	-1.12	0.9131

The Johansen and Julius λ_{trace} cointegration statistic test for testing the null hypothesis that there are at most *r* cointegrated vectors is used versus the alternative Hypothesis of more than *r* cointegrated vectors. Where: λ_{trace} is given by:

$$\lambda_{trace} = -T \quad {}_{\lambda=r+1}^{k} log(1-\lambda_i) \qquad (1)$$

Theory holds that two time series variables will be cointegrated if they have a long term or equilibrium relationship between them (Gujrati, 2003). Given that both series are I (1) imply that their linear combination is I (0). The maximum lag length was set to 7 quarters and an autoregressive order of p=6 were selected based on the partial correlation matrices and partial canonical matrices. The SAS procedure PROC VARMAX was used to test for cointegration and model fitting. The results of the cointegration tests are shown below (Table 2).

Tables 2a & 2b show the output from the VARMAX procedure based on the model specified (an intercept term is assumed). In the cointegration rank test using trace (Table 2a), we observe that there is no separate drift in the ECM and the process has a constant drift before differencing. These trace statistics are based on the alternate hypothesis (H_1) that there is a separate drift and no separate linear trend in the VECM.

The cointegration rank test using trace under restriction (Table 2b) shows the trace statistics based on the null hypothesis (H_0) that there is no separate drift in the VECM but a constant enters only via the error correction term.

In both cases the series are cointegrated with rank=1 because the trace statistics are smaller than the critical values. In the unrestricted case, Johansen’s trace statistic has a value of 16.076 which is greater than the 5% critical value of 15.34, therefore we reject r=0. Further, the test for r=1 versus r>1 does not reject r=1. Thus, Johansen’s test indicates a single (r=1) cointegrating vector.

The study proceeds to determine which result, either the model with restriction or the model with no restriction, is appropriate depending on the significance level. Since the cointegration rank is chosen to be 1 and the p-value is 0.0549, the hypothesis H_0 cannot be rejected at 5% significance level but can be rejected at the significance level of 10% (Table 3).

Since URate and CPI are cointegrated, according to the Granger representation theorem a cointegrated series can be represented by a vector error correction model (VECM) (Engle and Granger, 1987).

For H_0, a VECM (6) model with a *restriction on the deterministic term* will be used and a similar model with *no restriction* will be used for H_1.

Table 2a: Cointegration rank test using trace

H0:	H1:	**Eigenvalue**	**Trace**	**5% Critical Value**	**Drift in ECM**	**Drift in Process**
Rank=r	Rank>r					
0	0	0.2939	16.076	15.34	Constant	Linear
1	1	0.0723	2.8521	3.84		

Table 2b: Cointegration rank test using trace under restriction

H0:	H1:	**Eigenvalue**	**Trace**	**5% Critical Value**	**Drift in ECM**	**Drift in Process**
Rank=r	Rank>r					
0	0	0.4215	27.335	19.99	Constant	Constant
1	1	0.158	6.5366	9.13		

Table 3: Hypothesis test of the restriction

Rank	**Eigenvalue**	**Restricted Eigenvalue**	**DF**	**Chi-Square**	**Pr > ChiSq**
0	0.2939	0.4215	2	11.26	0.0036
1	0.0723	0.158	1	3.68	0.0549

Model with restriction on deterministic term

The univariate equations are found to be a good fit for the data based on the model F statistics and R-square statistics. The regression of Δ URate resulted in a model F test 3.7 and R-square of 0.64. Similarly the regression of Δ CPI resulted in a model F test of 4.3 and R-square of 0.674 (Table 4).

The residuals are checked for normality and autoregressive conditional heteroskedasticity or ARCH effects. The model also tests whether the residuals are correlated. The Durbin-Watson test statistics are both near 2 for both residual series and the series does not deviate from normal and are homoscedastic. The results also show that there are no ARCH effects on the residuals since the "no ARCH" hypothesis cannot be rejected given the F values (Table 5). There are no AR effects on the residuals - for both residual series the autoregressive model fit to the residuals show no significance indicating that the residuals are uncorrelated (Table 6).

Model with no restriction

The univariate equations are found to be a good fit for the data based on the model F statistics and R-square statistics. The regression of Δ URate resulted in a model F test 4.33 and R-square of 0.675. Similarly the regression of Δ CPI resulted in a model F test of 4.5 and R-square of 0.676 (Table 7).

The residuals are checked for normality and ARCH effects. The model also tests whether the residuals are correlated. The Durbin-Watson test statistics are both near 2 for both residual series and the series does not deviate from normal and are homoscedastic. The results also show that there are no ARCH effects on the residuals (Table 8). There are no AR effects on the residuals - for both residual series the autoregressive model fit to the residuals show no significance indicating that the residuals are uncorrelated (Table 9).

Table 4: Univariate model ANOVA diagnostics

Variable	R-Square	Standard Deviation	F Value	Pr > F
URate	0.6400	0.6644	3.7	0.0028
CPI	0.6735	0.5951	4.3	0.0010

Table 5: Univariate model white noise diagnostics

Variable	Durbin Watson	Normality		ARCH	
		Chi-Square	Pr > ChiSq	F Value	Pr > F
URate	1.9628	1.71	0.4256	1.12	0.2981
cpi	1.7549	1.27	0.5287	0.56	0.4585

Table 6: Univariate model AR diagnostics

Variable	AR1		AR2		AR3		AR4	
	F Value	Pr > F	F Value	Pr > F	F Value	Pr > F	F Value	Pr > F
URate	0	0.9733	0.01	0.9866	0.01	0.998	0.26	0.9
CPI	0.19	0.6673	0.07	0.9361	0.25	0.8621	0.41	0.803

Table 7: Univariate model ANOVA diagnostics

Variable	R-Square	Standard Deviation	F Value	Pr > F
URate	0.6751	0.6292	4.33	0.001
CPI	0.676	0.5927	4.35	0.0009

Table 8: Univariate model white noise diagnostics

Variable	Durbin Watson	Normality		ARCH	
		Chi-Square	Pr > ChiSq	F Value	Pr > F
URate	1.9978	2.55	0.2791	3.35	0.0757
CPI	1.7753	1.28	0.5262	0.53	0.4713

Table 9: Univariate model AR diagnostics

Variable	AR1		AR2		AR3		AR4	
	F Value	Pr > F	F Value	Pr > F	F Value	Pr > F	F Value	Pr > F
URate	0.06	0.8149	0.45	0.6418	0.26	0.8565	0.35	0.8426
CPI	0.11	0.7392	0.04	0.9634	0.25	0.8575	0.42	0.7894

Testing weak exogeneity

Results from the weak exogeneity test indicate that the unemployment rate (URrate) is a weak exogeneity of the consumer price index (CPI), whereas the CPI is not a weak exogeneity of URate (Table 10).

Table 10: Testing weak exogeneity of each variable

Variable	DF	Chi-Square	Pr > ChiSq
URate	1	1.94	0.164
CPI	1	10.02	0.0016

Conclusion

The expected result of the above analysis consists in a formal statistical confirmation of the existence of a unique linear and lagged relationship between inflation and unemployment for the period between September 2000 and June 2011. Hence, the two variables, being non-stationary I(1), are cointegrated ; i.e. their residual time series in the VECM representation has been proved to be stationary. This relationship can be explored further for the development of appropriate forecasting models incorporating other study variables.

References

Apergis, N. Theodosiou, L. (2008) "The employment-wage relationship: was Keynes right after al",*American review of political economy*,6(1):,40-50.

Engle, R. (2001) "GARCH 101: The Use of ARCH/GARCH Models in Applied Econometrics", *Journal of Economic Perpectives*,15(4),157-168.

Galizzi, M. (2005)"Wage changes, establishment growth, and the effect of composition bias"*Eastern economic journal*, 31(2).

Gujarati,D. (2003) *Basic Econometrics*,3rd ed, McGraw Hill Publishers.

Hendry, D. F. and Juselius, K. (2001) ,Explaining Cointegration Analysis: Part ii. Department of Economics, University of Copenhagen, Denmark.

Kitov, I. (2006b) Inflation, unemployment, labor force change in the USA, working papers 28, ECINEQ, society *for the study of economic inequality*.

Kitov, I. Kitov, O. Dolinskaya, S. (2007a) Relationship between inflation, unemployment and labor force change rate in France: cointegration test, MPRA Paper 2736, University library of Munich, Germany.

Kunst, M. R. (2009) Applied time series analysis-Part i.

Onder, K. Durgun, A. Unpublished.,Effects of tourism sector on the employment in Turkey: An econometric application.

Pramod, R. Unpublished,Time series forecasting of employment using SAS.

Stephanides, G. (2006) "Measuring the NAIRU: Evidence from the European Union, USA and Japan", *International Research journal of Finance and Economics*. 3.

Wassell,jr. C.S. Saunders, P.J. Unpublished, Time series on social security and private saving: The issue revised.

Yusof, S. A.(2007)"The long-run and dynamic behaviors of wages, productivity and employment in Malaysia", *Journal of Economic studies*, 35(3),249-262.