

## Is there a best kernel density estimator?

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Kernel density estimation has been a subject of many papers. A lot of research focused on constructing estimators that would reduce bias, relative to the ones suggested earlier. Thus, the argument in the previous papers was that "our estimator is better because it has a lower bias than that of some or all of the previous estimators". In this paper we construct kernel estimators that have arbitrarily small bias. This can be achieved without increasing the smoothness of the density, the kernel order or the variance of the constructed estimator. We also show that higher-order terms in the decompositions of bias and variance do not grow as bias becomes smaller.

Another issue addressed in the paper is testing if the density satisfies a certain differential equation. This provides a novel approach to normality testing.

Key Words: higher-order kernels, bias reduction, local normality test