

Special Coxian phase-type distributions discovered for modelling  
patient length of stay in hospital

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The Coxian phase-type distribution is a type of Markov model that considers a survival distribution as consisting of phases in a process which commence in a transient state or form, and move through successive states until finally terminating in a single absorbing state at the end (Neuts, 1981). Previous research has demonstrated the success of using the Coxian phase-type distribution to represent patient length of stay in hospital where patients commence the process in the first state and are absorbed in the terminating node when they leave hospital. In doing so, it has become apparent that the phase-type representation, using this statistical distribution, can directly map on to the different physical stages in the process of care of the patient in hospital. An example where this is the case (McClellan and Millard, 1993) is the modelling of elderly patient length of stay in hospital. It has previously been found through fitting the Coxian phase-type distribution to patient length of stay data, that a three phase Coxian mathematically provides the best representation of the data. When the three phase representation is proposed to the clinical team, it is confirmed that the patients generally have three phases of stay; acute, long-stay, rehabilitative stay agreeing with three stages of in the Markov process. The average length of time estimated for each phase also agrees with what the actual activity in the hospital ward.

Recent data for elderly patient stay in hospital has highlighted that a special form of the Coxian phase-type distribution is now more appropriate to represent patient survival. This paper aims to introduce this special type of Coxian phase-type distribution where patients all transit through certain phases referred to as tunnel phases. This latest form of the distribution more suitably represents hospital length of stay for recent data where it is possible that the hospital has more formally introduced a triage procedure or protocol for which all patients have to undergo.

#### References

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