Measure on dependent structure of financial markets under international diversification with application to emerging markets of the BRIC

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Abstracts

Analysis of the emerging financial markets dependent structure in the BRIC countries under international diversification is of great significance to multidimensional distribution assets. Considering the difficulty of the high-dimensional modeling,vine Copula will be used to mearsure the financial markets dependent structure.Then, the dependent structure of the BRIC stock markets are studied by vine Copula.From the results,the international diversification is the inevitable choice to conform to the capital international trend, and the BRIC stock markets can be used as the preferred markets in terms of the international diversified investment owing to the low correlation among them.

Keywords: international diversification,stock markets,vine copula

1. Introduction

The benefit of international diversification is of great importance in today's economic climate. Investors often assess diversification benefits in different countries or regions by international diversification. A growing number of studies have shown that the international diversification is an important means to improve the return on assets(Samuelson(1967), Jonathan and Fabrizio(2011), Lorán et al.(2011)).

It is vital to have accurate measures of dependence. According to Markowitz (1952), the income will not be higher even when the assets more dispersed. On the contrary, investors' income will increase significantly when the correlation between the international financial markets is weak. Therefore, the benefits and risks in the international diversification of investment is closely related to the correlation or the dependent structure of financial markets.

Diversification is assessed with various dependence measures. In light of the above discussion, we estimate dependence in two ways, using correlations and copulas. In recent years copula modeling has become increasingly popular in finance.Cpula may possess complex dependence structures—linear,nonlinear,and tail dependence —and this had raised questions and highlighted concerns about the appropriateness of the multivariate normal assumption and the use of the correlation coefficient. For the bivariate case,a rich variety of copula families is available and well-investigated (Nelsen,2007).However, in arbitrary dimension, the choice of adequate families is rather limited. Standard multivariate copulas such as the multivariate Gaussian or Student-t as well as exchangeable Archimedean copulas lack the flexibility of accurately modeling the dependence among larger numbers of variables. Generalizations of these offer some improvement, but typically become rather intricate in their structure and hence exhibit other limitations such as parameter restrictions. Therefore, we need to use multivariate copulas to study the correlation between the dependent structure of financial markets.

BRIC (Brazil, Russia, India and China) are viewed currently as pillars of relative political, economic and financial stability, with the prospect of a major shift in future world power(Haifeng Xu and Hamori,2012). They are committed to advance "the reform of international financial institutions", have pledged to work together on "political and economic issues such as energy and food security" and to cooperate to promote "fundamental research and the development of advanced technologies." The

BRIC together currently account for more than a quarter of the world's land area, more than 40% of the world's population and about 15% of global GDP. At the trend of expanded international diversification,the BRIC's proportion of the international investment portfolio is increasing. Therefore, we need a more in-depth study on the dependent structure of financial markets in the BRIC countries.

In this paper, we model the dependent structure of BRIC's financial data using the concept of vine Copula. We select C/D-vine copula and identify the type of dependence captured by each country. The remaining parts of the paper are organized as follows: Section 2 provides a brief review of the vine Copula and Section 3 describes the selecting of vine Copula models. Section 4 presents the empirical results. Finally,Section 5 provides the conclusions.

2. Vine Copulas

Vine copulas do not suffer from any of these problems. Initially proposed by Joe (1996) and developed in more detail in Bedford and Cooke (2001, 2002) and in Kurowicka and Cooke (2006), vines are a flexible graphical model for describing multivariate copulas built up using a cascade of bivariate copulas, so-called pair-copulas. Their "statistical breakthrough" was due to Aas et al. (2009) who described statistical inference techniques for the two classes of canonical (C-) and D-vines.

These are derived as iterative pair-copula constructions, where the *d(d-1)/2* pair-copulas can be arranged in *d-1* trees (acyclic connected graphs with nodes and edges). In the first C-vine tree, the dependence with respect to one particular variable, the first root node, is modeled using bivariate copulas for each pair. Conditioned on this variable, pairwise dependencies with respect to a second variable are modeled, the second root node. In general, a root node is chosen in each tree and all pairwise dependencies with respect to this node are modeled conditioned on all previous root nodes, i.e., C-vine trees have a star structure. This gives the following decomposition of a multivariate density, the C-vine density w.l.o.g. root nodes 1,…,d (otherwise nodes can be relabeled),

$$f(x_1,\mathbf{K},x_d)=\prod_{k=1}^{n} f(x_k)\times\prod_{j=1}^{n-1}\prod_{i=1}^{n-j} c_{j,j+1|1,\mathbf{K},j-1}\left(F\left(x_j\,\middle|\,x_1,\mathbf{K},x_{j-1}\right),F\left(x_{j+i}\,\middle|\,x_1,\mathbf{K},x_{j-1}\right)\right) \tag{1}$$

where f_k,k=1,…,d,denote the marginal densities and $c_{j,j+1}$ bivariate copula densities.

Similarly, D-vines are also constructed by choosing a specific order of the variables. Then in the first tree, the dependence of the first and second variable, of the second and third, of the third and fourth, and so on, is modeled using pair-copulas, i.e., if we assume the order 1,...,d, we model the pairs (1,2), (2,3), (3,4), etc. In the second tree, conditional dependence of the first and third given the second variable (the pair (1,3 | 2)), the second and fourth given the third (the pair (2,4 | 3)), and so on, is modeled. In the same way, pairwise dependencies of variables a and b are modeled in subsequent trees conditioned on those variables which lie between the variables a and b in the first tree, e.g., the pair (1,5 | 2,3,4). That is each D-vine tree has a path structure. This then leads to the D-vine density which also conveniently decomposes a d-dimensional density (as above the order is w.l.o.g. chosen as 1,...,d; otherwise nodes can be relabeled):

$$f(x_1,\mathbf{K},x_d)=\prod_{k=1}^{n} f(x_k)\times\prod_{j=1}^{n-1}\prod_{i=1}^{n-j} c_{j,i+j|i+1,\mathbf{K},i+j-1}\left(F\left(x_i\,\middle|\,x_{i+1},\mathbf{K},x_{i+j-1}\right),F\left(x_{i+j}\,\middle|\,x_{i+1},\mathbf{K},x_{i+j-1}\right)\right) \tag{2}$$

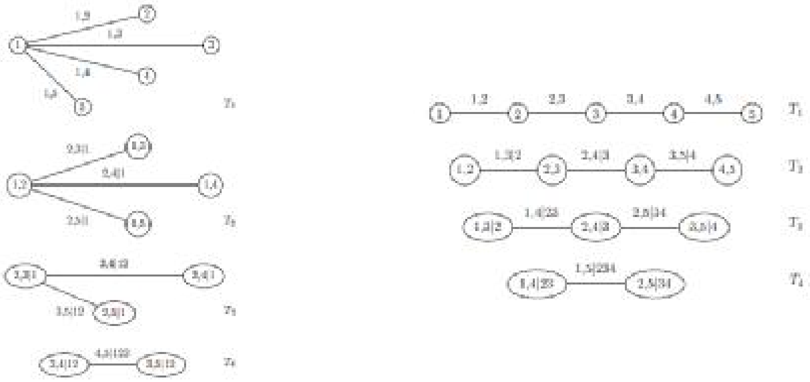

Figure 1: Examples of five-dimensional C- (left panel) and D-vine trees (right panel) with edge indices.

By allowing arbitrary bivariate copulas for each pair-copula term in the decompositions (1) and (2), the multivariate copulas obtained from C- and D-vine structures, so-called C- and D-vine copulas, constitute very flexible models, since bivariate copulas can easily accommodate complex dependence structures such as asymmetric dependence or strong joint tail behavior (Joe, Li, and Nikoloulopoulos 2010). Examples of five-dimensional C- and D-vine trees are shown in Figure 1. Here, the order of root nodes in the C-vine is 1,...,5, which also is the order of the first D-vine tree. Edge labels show the indices of the corresponding pair-copula terms.

3. Selection of vine copula models

As already noted there are many different orderings of the variables in C-vine models possible. We will now consider these selection problems.As noted in Aas et al. (2009) it is preferable to choose models with high dependence in the bivariate conditional distribution characterized by $c_{i,j|i_1,\dots,i_k}$, where the number of conditioning variables k is small. This suggests a data driven sequential approach starting with determining the d−1 unconditional pair-copulas needed in a C-vine copula. For this estimate all pairwise Kendall's $\tau_{i,j}$ values by $\tau^*_{i,j}$ and find the variable i^* which maximizes

$$\hat{S}_i := \sum_{j=1}^{d} \left| t^*_{i,j} \right| \tag{3}$$

over i = 1,…, d. Here we set $\tau^*_{i,i} = 1$ for i = 1, . . . , d. To ease notation we reorder the variables in such a way that the first variable is now i^*. For this reordering $c_{1,j+1}$, j = 1,…,d−1 are selected as unconditional pair-copulas.We call variable 1 also the root of all unconditional pair-copulas. Before determining the pair-copulas with the single conditioning variable 1, a choice of the pair-copula family and its parameter value for $c_{1,j+1}$ for j = 1,…,d−1 has to be made.

We now consider the problem of choosing the copula family. This has been a well studied problem and many procedures have been suggested. Note that for the sequential selection procedure we only require a copula selection in two dimensions. Copula goodness-of-fit tests have been studied by Genest et al. (2009) and Berg (2009).

Joe (1997) proposed a model selection based on the Akaike information criterion (AIC) of Akaike (1974).

$$AIC := -2\sum_{i=1}^{n} \log f(x_i; q^*) + 2k \tag{4}$$

where θ^* denotes the estimate of θ and k is the number of parameter $\theta=(\theta_1,\dots,\theta_k)^T$ in the model. Specifying the AIC to a specific copula with density c we get

$$AIC := -2\sum_{i=1}^{n} \log c(u_{i1}, u_{i2}; q^*) + 2k \tag{5}$$

which can be used as a copula selection criterion. The advantage of this selection method is that it can be automatized in a copula selection program.

Next we describe how the parameters of the C-vine density given by (1) or D-vine density given by (2) can be estimated by maximum likelihood. Inference for a vine is also feasible, but the algorithm is not as straightforward.

we will use the function $h(x, v, \theta)$ to represent this conditional distribution function when x and v are uniform, i.e. $f(x) = f(v) = 1$, $F(x) = x$ and $F(v) = v$. That is,

$$h(x, v, q) = F(x|v) = \frac{\partial C_{x,v}(x, v, q)}{\partial v} \tag{6}$$

where the second parameter of $h(\cdot)$ always corresponds to the conditioning variable and θ denotes the set of parameters for the copula of the joint distribution function of x and v.

Starting values of the parameters needed in the numerical maximisation of the log-likelihood may be determined as follows:

(1) Estimate the parameters of the copula in tree 1 from the original data;

(2) Compute observations (i.e. conditional distrubution functions) for tree 2 using the copula parameters from tree 1 and the h(·) function;

(3) Estimate the parameters of the copulae in tree 2 using the observations from (2);

(4) Repeat the second step and the third step until the parameters of the last tree estimated.

Finally, how to choose the best model between C-vine and D-vine Copula? We will use the Vuong and the Clarke tests to select the best model(Vuong,1989). These tests are suitable to compare two models, which are non-nested. Both are likelihood ratio based and related to the common Kullback-Leibler information criterion, which measures the distance between two statistical models.

4. Applications

We apply now our C-vine and D-vine model to 4 time series of stock market weekly return from BRIC from June 12, 2002 until September 24, 2012. Therefore we have 520 values available for each country considered. For simplification we use the following abbreviations: CHN(China), RUS (Russia), BRA(Brazil) and IN (India).

Before analyzing the dependence in the data set, we selected appropriate time series models for the univariate margins. For the stock returns, we used an AR(1)-model or an ARMA(1,1)-model. Further, for all variables, a GARCH(1,1)-model was used to model the volatility. The error distribution of the GARCH-model was chosen to be the Student's t-distribution.After filtering the original returns with the chosen univariate models, the standardized residual vectors are converted to uniform pseudo-observations using their empirical distribution functions.

Table 1: Empirical Kendall's τ matrix and the sum over the absolute entries of each row

	BRA	CHN	IN	RUS	S
BRA	1.000	.152	.350	.327	1.829
CHN	.152	1.000	.146	.105	1.403
IN	.350	.146	1.000	.272	1.768
RUS	.327	.105	.272	1.000	1.704

We apply now the sequential procedure to select an appropriate vine copula for the stock market copula data. Table 1 gives the empirical Kendall's τ matrix and the sum of their absolute values, denoted by S. From this we see that BRA is the first root variable. Given this first root variable and the sequential vine identification procedure from previous chapter the next root variable CHN followed by IN and finally RUS can be identified.

For our implementation for the copula family choice, we consider the Gaussian,t-, Clayton, Gumbel, Frank, BB1 and BB7 copula family, which cover a wide range of dependence behavior.

Table 2: Estimates for C-vine and D-vine copula models

	C-vine				D-vine			
i	Parameter	Copula	θ	AIC	Parameter	Copula	θ	AIC
0	$\rho_{BRA,IND}$	t	0.519	-166.72	$\rho_{BRA,IND}$	t	0.519	-166.72
	$\nu_{BRA,IND}$		5.018		$\nu_{BRA,IND}$		5.018	
	$\rho_{BRA,RUS}$	BB7	1.334	-168.59	$\rho_{IND,RUS}$	Survial-BB7	1.325	-117.67
	$\nu_{BRA,RUS}$		0.652		$\nu_{IND,RUS}$		0.417	
	$\rho_{BRA,CN}$	t	0.241	-36.86	$\rho_{RUS,CN}$	t	0.163	-19.14
	$\nu_{BRA,CN}$		5.549		$\nu_{RUS,CN}$		5.544	
1	$\rho_{IND,RUS \mid BRA}$	t	0.216	-28.89	$\rho_{BRA,RUS \mid IND}$	BB1	0.321	-77.23
	$\nu_{IND,RUS \mid BRA}$		7.181		$\nu_{BRA,RUS \mid IND}$		1.132	
	$\theta_{IND,CN \mid BRA}$	Survial- Gumbel	1.073	-5.33	$\theta_{IND,CN \mid RUS}$	Survial-Gumbel	1.123	-17.92
2	$\rho_{RUS,CN \mid BRA,IND}$	t	0.016	0.64	$\theta_{BRA,CN \mid IND, RUS}$	Frank	0.921	-9.50
	$\nu_{RUS,CN \mid Brazil,IND}$		11.883					
	AIC	-405.7406			-408.1872			

The chosen copula types and estimated parameters for the all trees are given in Table 2.Note that the variable i indicates the number of variables in the conditioning set and the pair-copula family type chosen.

Having fitted the full C-vines and D-vines, in order to determine the better fitting vine copula model for the BRIC stock markets, we perform a Vuong test comparing both models. Table 3 shows the results.The test statistics close to zero (irrespective of the correction considered) and the *p* values indicate that the C-vine copula model is better fitting model.

Table 3: Test statistics of Vuong test

Vuong	0.05068923	vuong.Schwarz	-0.7603768
P	0.9595732	vuong$p.value.Schwarz	0.4470294

5. Conclusions

In this paper we introduced the class of vine copulas and provided effective estimation procedures for the unknown parameters. From the results,the international diversification is the inevitable choice to conform to the capital international trend, and the BRIC stock markets can be used as the preferred markets in terms of the international diversified investment owing to the low correlation among them.

To summarize, the above analysis showed low positive dependencies among the BRIC stock indices, where the Brazil stock market was determined to be central for explaining the overall dependence observed in the data. Based on the data we could discriminate among fitted C- and D-vine copula models, where it should be noted that both models provide additional insights due to their specific structures.

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