Matrix Transformation Technique based Forecast Modeling of Input-Output Table

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Abstract

Given its importance, recent decades have witnessed an increasing growth in literatures concerning application of input-output tables. However, the problem of too-long interval time and time delay when constructing input-output tables has always existed. In order to improve the timeliness of input-output tables and forecast the structural changes of input-output table, we present an MTT method for serial input-output tables forecasting mainly based on matrix transformation techniques. The method releases the constraints of input-output tables, obtains a series of unconstrained matrices, forecasts the unconstrained matrices based on the law of historical data, and eventually gets forecasted input-output tables through the inverse computation of matrix transformation. An empirical analysis applied on the data provided by Miller and Blair (1985) has demonstrated the merit of MTT.

Key words: constraints; empirical analysis; forecast; matrix transformation; input-output tables

1. Introduction

Known as the core of input-output technique (Leontief, 1936), input-output table exhibits a thorough statistical overview to interactions in an economy. There exists, however, a problem of timeliness of input-output table, which seriously restricted its broad use. The majority of national statistical institutes provide benchmark tables based on detailed surveys only at five year intervals, which leads to the intervals of serial input-output tables are too long. It is always statisticians and economists' key focus areas to constructing benchmark input-output tables, including interpolating and extrapolating, to produce new estimates for analytical use.

Up till recent, many different non-survey methods have been proposed and extensively used in updating the input-output tables. The major difference exists in that each method is based on either statistics or optimization techniques. Generally speaking, the family of statistical methods consists of naive method (Benjamin and Ahmadi-Esfahani, 1998), RAS method (Stone and Brown, 1962), and GRAS algorithm (Junius and Oosterhaven, 2002). The basic idea of optimization methods lies in minimizing dissimilarity between the prior table and the target table to be estimated, with a strict constraint that economic data, including the gross outputs, intermediate output totals and intermediate input totals by industries in the target years, are known. The pioneering works were contributed by Friedlander (1961) and Matuszewski et al. (1964), who respectively proposed the approach of Normalized Squared Differences (NSD) and Normalized Absolute Differences (NAD). Jackson and Murray (2004) provided recent overviews and evaluations of various methods.

Moreover, they proposed several alternative least-square minimands, which all minimize the distance between the target tables and the prior tables.

Despite the vast amount of expert endeavor, there are still many challenging issues which have not been fully addressed in previous literatures. For instance, almost all of the existing methods rely on the assumption that the transaction coefficients won't fluctuate too much during the forecast period, which indeed may not hold on the condition that economy structure changes (Israilevich et al., 1997). Moreover, the existing methods cannot proceed without an access to the economic data of gross outputs, intermediate output totals and intermediate input totals by industries in the target years. Undoubtedly, this again limits their applications for input-output table forecasting after current year, when the aforementioned economic data are unknown yet. Finally, optimization methods are always computational complex and sometimes even don't have optimal solutions. Given these facts, it's still a problem needed further study to forecast the target years' input-output tables according to the known sequence of input-output tables.

To this end, this paper intends to investigate a novel method for forecasting input-output tables, referred to as Matrix-Transformation-Technique-based forecast method (MTT). High emphasis shall be first placed on that our MTT method directly uses the input-output tables, instead of the corresponding transaction coefficient tables, for forecasting. To be specific, by using the technique of matrix transformation and its inverse computation, MTT releases constraints associated with original input-output matrices, and balance the unconstrained estimated matrices towards the corresponding input-output matrices with internal coherence. Indeed, this has contributed to successfully shake off the strict assumption of little-fluctuated transaction coefficients. Besides, with a sequence of unconstrained input-output matrices, MTT enables forecasting the target year's input-output table according to classic techniques of time series analysis, no matter positive or negative entries in the matrices. Another remarkable advantage lies in that MTT could still proceed regardless of availability of the economic data by industries in target years. When candidate years come after the current year, the unavailable economic data can even be endogenously derived by MTT method. In addition, the calculation of MTT is much simple and convenient for handling in practice. In a word, the procedure of MTT concerns on the rule of industrial structure's gradual changes, looks for economic interpretation of input-output tables' adjustment, and analyzes the dynamic trend of input–output table time series. To demonstrate the merits of MTT method, an empirical analysis based on US input-output tables (Miller and Blair, 1985) has been conducted and MTT has displayed its superiority in forecasting accuracy.

The remainder is organized as follows. In Section 2, we develop the MTT method. Then, in Section 3, empirical evaluation of the method is proposed. Finally, in Section 4, we draw conclusions.

2. Model

In this section, we first give a brief introduction to techniques of matrix transformation. Also, we put forward a novel method for forecast modeling for input-output matrix.

2.1. Preliminaries

An *n*-sector input-output table can be simplified as input-output matrix $\mathbf{X}$, i.e.,

$$\boldsymbol{X}=\begin{bmatrix} x_{1\,1} & \cdots & x_{n1} & x_{1(n+1)} \\ \vdots & \ddots & \vdots & \vdots \\ x_{n1} & \cdots & x_{nn} & x_{n(n+1)} \\ x_{(n+1)1} & \cdots & x_{(n+1)n} & x_{(n+1)(n+1)} \end{bmatrix}_{(n+1)\times(n+1)}, \qquad (1)$$

where $x_{ij}\,(i=1,\cdots,n,\ j=1,\cdots,n)$ are the sectoral intermediate inputs and outputs; $x_{i(n+1)}\left(i=1,2,\cdots,n\right)$ is total final demand of Sector *i* and $x_{(n+1)j}\left(j=1,2,\cdots,n\right)$ is total added value of Sector *j*. To facilitate the derivation, we make $x_{(n+1)(n+1)}$ as a constant in order to facilitate the derivation, i.e., $x_{(n+1)(n+1)}=\sum_{i=1}^{n}x_{i(n+1)}=\sum_{j=1}^{n}x_{(n+1)j}$.

Hence, the problem of input-output table forecasting is converted to - Given the history years' input-output matrices $\mathbf{X}^1,\mathbf{X}^2,\cdots,\mathbf{X}^T$, forecast matrices $\hat{\mathbf{X}}^{T+1},\cdots,\hat{\mathbf{X}}^{T+L}\left(L\geq 1\right)$ in target years.

2.2. Techniques of matrix transformation

According to the characteristics of input-output tables, the elements in matrix $\mathbf{X}$ need to satisfy the following constraint, $\sum_{i=1}^{n+1}x_{ik}=\sum_{j=1}^{n+1}x_{kj}$, $k=1,\cdots,n+1$.

Evidently, the constraint inherent to matrix $\mathbf{X}$ is that the summation of each row is equal to the corresponding column summation. To facilitate subsequent derivation, we **transform** $\mathbf{X}$ by $y_{ij}=x_{ij}\big/x_{i(n+1)}$, $i,j=1,2,\cdots,\left(n+1\right)$, which releases constraints in matrix $\mathbf{X}$. Since each sector's total final demand and the summation of all the sectors' added values are always positive, i.e., $x_{i(n+1)}\neq 0\ (i=1,2,\cdots,n)$ and $x_{(n+1)(n+1)}\neq 0$, the transformation above will always proceed.

Now, the matrix $\mathbf{X}$ turns into

$$\mathbf{Y}=\begin{bmatrix} y_{11} & \cdots & y_{1n} & 1 \\ \vdots & \ddots & \vdots & \vdots \\ y_{n1} & \cdots & y_{nn} & 1 \\ y_{(n+1)1} & \cdots & y_{(n+1)n} & 1 \end{bmatrix}_{(n+1)(n+1)}. \qquad (2)$$

Remark. The economic interpretation of $y_{ij}\left(i,j=1,2,\cdots,n\right)$ in matrix $\mathbf{Y}$ is the proportion of the monetary value of the transactions between a pair of sectors (from Sector *i* to Sector *j*) to the total final demand of Sector *i*. And $y_{(n+1)j}\left(j=1,2,\cdots,n\right)$ concerns the ratio of Sector *j*'s added value accounting for the summation of all the sectors' added values.

Next, we will discuss how to get matrix $\mathbf{X}$ when known $\mathbf{Y}$. The process of **inverse computation** is summarized as follows:

Step 1: For $j=1,2,\cdots,n$, we can calculate elements in the last row of $\mathbf{X}$ according to $x_{(n+1)j} = y_{(n+1)j} \cdot x_{(n+1)(n+1)}$.

Step 2: Given $x_{(n+1)j}\left(j=1,2,\cdots,n\right)$, the elements in the last column of $\mathbf{X}$, i.e., $x_{k(n+1)}\left(k=1,2,\cdots,n\right)$, can be derived by $\sum_{j=1}^{n}\left(y_{kj}+1\right)x_{k(n+1)} - \sum_{i=1}^{n} y_{ik}x_{i(n+1)} = x_{(n+1)k}$, which comes from the transformation equation and the first n formulas in constraint equation. What's more, note that $x_{(n+1)k} = y_{(n+1)k}x_{(n+1)(n+1)}$, we could rewrite the formulas by matrix form $\boldsymbol{BX}_{\bullet(n+1)} = \boldsymbol{X}_{(n+1)\bullet}$, where $\mathbf{X}_{(n+1)\bullet} = \left[x_{(n+1)1}, x_{(n+1)2}, \cdots, x_{(n+1)n}\right]'$, $\mathbf{X}_{\bullet(n+1)} = \left[x_{1(n+1)}, x_{2(n+1)}, \cdots, x_{n(n+1)}\right]'$, and matrix $\boldsymbol{B}$ is given by

$$\boldsymbol{B} = \left(\begin{bmatrix} 1+\sum_{j=1}^{n} y_{1j} & 0 & \cdots & 0 \\ 0 & 1+\sum_{j=1}^{n} y_{2j} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1+\sum_{j=1}^{n} y_{nj} \end{bmatrix} - \begin{bmatrix} y_{11} & y_{21} & \cdots & y_{n1} \\ y_{12} & y_{22} & \cdots & y_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ y_{1n} & y_{2n} & \cdots & y_{nn} \end{bmatrix}\right). \qquad (3)$$

Evidently, $\boldsymbol{B}$ is a invertible matrix, thereby the solution of $\boldsymbol{X}_{\bullet(n+1)}$ can be given by $\boldsymbol{X}_{\bullet(n+1)} = \boldsymbol{B}^{-1}\boldsymbol{X}_{(n+1)\bullet}$.

Step 3: Given $\boldsymbol{X}_{\bullet(n+1)}$ obtained in Step 2, elements in matrix $\mathbf{X}$, i.e., $x_{ij}\left(i,j=1,2,\cdots,n\right)$ can be calculated by $x_{ij} = y_{ij}x_{i(n+1)}$.

Now, all the entries $x_{ij}\,(i,j=1,\cdots,n+1)$ in matrix $\mathbf{X}$ have been calculated from matrix $\mathbf{Y}$ through the above three steps.

2.3. The input-output matrix forecast modeling

Equipped with the aforementioned notations, we hereby put forward a novel method for forecast modeling for input-output matrix. Since the method is heavily relies on the **Matrix Transformation Technique**, we refer it to as **MTT** in what follows.

1. Transformation Stage: For $t=1,2,\cdots,T$, transform $\mathbf{X}^t$ into $\mathbf{Y}^t$ by using transformation equation.

2. Forecasting Stage: Given a sequence of matrices $\mathbf{Y}^1, \mathbf{Y}^2, \cdots, \mathbf{Y}^T$, forecast $\hat{\mathbf{Y}}^{T+l}$.

a) For $i=1,2,\cdots,n$ and $j=1,2,\cdots,n$, establish forecasting models for y_{ij}^t, i.e.,

$y_{ij}^t = f_{ij}(t) + \varepsilon_{ij}$. Taking $t = T + l$ as the target year, the model enables us to forecast $\hat{y}_{ij}^{T+l}(i, j = 1, 2, \cdots, n)$ in matrix $\hat{\mathbf{Y}}^{T+l}$.

b) Assuming that is non-zero, the vector $\left[y_{(n+1)1}^t, y_{(n+1)2}^t, \cdots, y_{(n+1)n}^t\right]'$ can be transformed into $\left[z_1^t, z_2^t, \cdots, z_{n-1}^t, 1\right]'$, where for $j = 1, 2, \cdots, n$, $z_j^t = y_{(n+1)j}^t / y_{(n+1)n}^t$. Since the vector of $\left[z_1^t, z_2^t, \cdots, z_{n-1}^t, 1\right]'$ has shaken off the unit-sum constraint associated with $\left[y_{(n+1)1}^t, y_{(n+1)2}^t, \cdots, y_{(n+1)n}^t\right]'$, we are now free to choose forecasting model for $z_j^t\left(j = 1, 2, \cdots, n-1\right)$, i.e., $z_j^t = g_j(t) + \delta_j$. Taking $t = T + l$, we use the function of $g_j(t)$ to calculate $\hat{z}_j^{T+l}$. Finally, back-transform $\hat{z}_j^{T+l}$ into $\hat{y}_j^{T+l}$ by

$$\begin{cases} \hat{y}_{(n+1)j}^{T+l} = \dfrac{\hat{z}_j^{T+l}}{1 + \sum_{j=1}^{n-1} \hat{z}_j^{T+l}}, \quad j = 1, 2, \cdots, n-1 \\ \hat{y}_{(n+1)n}^{T+l} = \dfrac{1}{1 + \sum_{j=1}^{n-1} \hat{z}_j^{T+l}} \end{cases} . \tag{4}$$

3. Inverse-Transformation Stage: Given $\hat{\mathbf{Y}}^{T+l}$, estimate input-output matrices $\hat{\mathbf{X}}^{T+l}$ by inverse computation.

3. Empirical assessment

In this section, we present the empirical evaluation results of MTT method. To assess the performance of input-output table forecast model, we use three evaluation indexes: *Standardized Total Percentage Error* (*STPE*: Miller and Blair, 1985), *Theil's U* (Theil, 1971), *C index* (Roy et al，1982), whose basic ideas are all measuring the similarity between input-output matrix actual value and the estimated value. In all instances, the smaller the three criteria are, the higher the forecast accuracy will be.

To accomplish empirical applications, the 23-sector US input-output tables in years 1967, 1972 and 1977 drawn from Miller and Blair (1985) are used in this analysis. Our empirical analysis will concentrate on forecasting 1977 table on the basis of dynamic data trend extracted from both 1967 and 1972 tables. In the modeling process, we assume that the change rates of the entries in unconstrained matrix are uniform, namely, their growth rates remain constant from 1967 to 1977. Also, the added values and final demands of each sector in target year 1977 are assumed unknown. As indicated by Table 1, the performance of MTT method is satisfactory with respect to the concerning criterions. The result has unveiled the merits of MTT method in forecasting input-output tables after current fiscal year.

Table 1 Results of forecasting input-output table of 1977 for US by MTT method

Criteria	*STPE*	*U*	*C*
Values	18.158	0.128	0.026

4. Conclusions

In this paper, we propose a matrix-transformation-technique-based MTT method to forecast input-output tables. The key idea is to release constraints associated with the entries of input-output matrix by the matrix transformation technique. Given a series of unconstrained matrices, obtained by matrix transformation techniques, we can apply MTT method to forecast the input-output tables in the period from the latest benchmark year to the current fiscal year.

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