Specifying Asymmetric STAR models with Linear and Nonlinear GARCH Innovations: Monte Carlo Approach

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Abstract

Economic and finance time series are mostly asymmetric and are expected to be modeled using asymmetrical nonlinear time series models. Smooth Transition Autoregressive (STAR) models: Logistic (LSTAR) and Exponential (ESTAR) are known to be asymmetric and symmetric respectively, and these have gained their popularity in empirical econometric modelling. Estimating only the LSTAR model for economic and finance data may give models that are not well diagnosed. Under non-normal and heteroscedastic innovations, the residuals of these models are estimated using Generalized Autoregressive Conditionally Heteroscedastic (GARCH) models with variants which include linear and nonlinear forms. The small sample properties of these STAR-GARCH variants are yet to be established but these properties are investigated using Monte Carlo (MC) simulation approach. An MC investigation was conducted to investigate the performance of selections of STAR-GARCH models by the classical nonlinear selection approaches. The ARCH(1) and GARCH(1,1) were the linear GARCH while the Logistic Smooth Transition-ARCH (LST-ARCH(1,1)), Logistic Smooth Transition-GARCH (LST-GARCH(1,1)) and Asymmetric Nonlinear Smooth Transition-GARCH (ANST-GARCH(1,1)) were the nonlinear GARCH. The nonlinearity parameter in the variance equations and Autoregressive (AR) parameters were varied along with different sample sizes. With the assumption of normality, the results showed that the selection of LSTAR models were actually affected by the structure of the innovations and this improved as sample size increased. Misspecification tests actually showed that these models cannot be misrepresents in the real sense.

Keywords: Asymmetry, Monte Carlo simulations, GARCH, Smooth Transition Autoregression, Specification

1. Introduction

Smooth Transition autoregressive (STAR) and Generalized Conditionally Heteroscedastic (GARCH) models are gaining their popularities in economics and finance. STAR models of Granger and Terásvirta (1993) classify market into two phases of recession and expansion, whereas GARCH model of Bollerslev (1986) is often used to study the behaviour of asset returns or innovations of the ‘parent’ model. In that case, such a ‘parent’ model is the mean equation and the volatility (GARCH) model is the variance equation. The innovations of the STAR model are expected follow normal distribution (homoscedasticity) but in case this is not true, the innovations are said to possess heteroscedasticity, which can be of various forms (Pavlidis, Paya and Peel, 2010). The ‘mean-variance’ equations are then compounded as STAR-GARCH model.

Maximum Likelihood Estimation (MLE) of STAR-GARCH model is examined in Chan and McAleer (2002). The structural and statistical properties of the model are also established in the paper, even though the asymptotic normality and finite sample properties are still examined using Monte Carlo simulation approach. Chan and McAleer (2002) also consider the effects of misspecifying the transition functions (logistic or exponential) in the STAR model and the results obtained showed that greater bias will be induced in the GARCH estimates for the STAR-GARCH model whenever STAR mode is misspecified. Their results further showed that Logistic STAR model can easily be substituted for Exponential STAR model.

This present paper is motivated by the work of Chan and McAleer (2002). We consider here the linear GARCH and Smooth Transition specification of ARCH/GARCH models in a Monte Carlo simulation approach. Nonlinearities were first introduced in the ARCH functional form in Engle and Bollerslev (1986). They proposed in their model the dynamics of conditional variance, \( \sigma_t^2 \) as it changes with the squared residuals and the transition between different conditional variance determined by normal cumulative distribution function. Few years later, Higgins and Bera (1992) developed a Nonlinear ARCH (NARCH) model which accommodated different functional forms to predict the conditional variance. Apart from the classical ARCH and GARCH models of Engle (1982) and Bollerslev (1986), Smooth Transition GARCH (STARCH), Smooth Transition GARCH (ST-GARCH) and Asymmetric Nonlinear Smooth Transition GARCH (ANST-GARCH) models of Hagerud (1996), Gonzalez-Rivera (1998) and Anderson et al. (1999) are also considered. The rests of the paper are structured as: Section 2 reviews the STAR-GARCH and STAR-STGARCH processes. Section 3 presents the Monte Carlo simulations experiment and the discussion of the results while Section 4 renders the concluding remarks.
2. The STAR-GARCH and STAR-STGARCH Models

This paper presents compounded regime switching and volatility models, with the regime switching model as the mean equation and volatility models as the variance equation. For a time series $y_t, t = 1, \ldots, N$ with $y_t \sim N(\mu, \sigma^2)$ in the structural model,

$$\hat{y}_t = f(\cdot) + \epsilon_t$$

(1)

where $f(\cdot)$ is the function of $y_t$ and $\epsilon_t$ is the innovations process, the innovations $\epsilon_t$ are expected to be independently and identically distributed with mean 0 and variance 1 that is homoscedasticity case. In the case where this assumption of normal distribution fails, the innovations are estimated with volatility models.

2.1 The Mean Equation: STAR model

The Smooth Transition Autoregressive (STAR) model is introduced in Granger and Teräsvirta (1993) and the specification, estimation and evaluation of the model are itemized following standard procedures in Teräsvirta (1994). Since then, the model has been applied to study nonlinearity in business cycle (Terasvirta and Anderson, 1992), Skalin and Terasvirta 1996; 1998) and real exchange rates (Baum et al., 1998; Liew et al., 2002). The connection between business cycle-regimes and nonlinearity in the UK labour market is studied in Acemoglu and Scotts (1994). Ocal (2000) applied STAR model on the nonlinearities in growth rates of some selected UK macroeconomic time series and suggest either two-regime or three-regime model for UK economy. Mourell, Cuestas and Gil-Alana (2011), Shittu and Yaya (2011) and Yaya (2013) considered STAR model for Nigerian inflation series.

Apart from real life time series data that has been considered so far for STAR model, Escribano and Jordà (2001) and Yaya and Shittu (2011) investigated the selection of STAR model by varying some of the parameters and conditions in the models and obtained results that serve as guide for nonlinear time series modelers. Then, there is need to study, and if possible develop the structural and small sample properties of the STAR model. The STAR model of order $p$ is given as,

$$y_t = \phi_{01} + \sum_{i=1}^{p} \phi_{1i} y_{t-i} + \left( \phi_{20} + \sum_{i=1}^{p} \phi_{2i} y_{t-i} \right) F\left(y_{t-d}; \gamma, c\right) + \epsilon_t$$

(2)

where $\phi_{01}, \phi_{20}$ are the constants and $\phi_{1i}, \phi_{2i} (i = 1, \ldots, p)$ are the autoregressive parameters of order $p$. The transition function, $F\left(y_{t-d}; \gamma, c\right)$ causes the nonlinear dynamics in the model, and this are of logistic and exponential forms as given as,

$$F\left(y_{t-d}; \gamma, c\right) = \frac{1}{1 + \exp \left[ -\gamma \left( y_{t-d} - c \right) \right]}$$

(3)

and

$$F\left(y_{t-d}; \gamma, c\right) = 1 - \exp \left[ -\gamma \left( y_{t-d} - c \right)^2 \right]$$

(4)

respectively, with $\gamma > 0$ in both cases. The logistic type is known to be asymmetric whereas the exponential type is symmetric. Economic and finance series often exhibit forms of asymmetries, and therefore Logistic STAR (LSTAR) model is often applied to model nonlinear dynamics in the series. In the transition functions, the transition variable is $y_{t-d}$ with $d$ assuming values 1, 2, ..., $p$. The value of $d$ is varied in order to improve nonlinearity in the system when it is not known prior to model estimation. The slope, $\gamma$ and intercept, $c$ are parts of the nonlinearity parameters in the transition function. As $\gamma$ assumes values from 1 to say 100, the nonlinearity becomes sharper, and the dynamics shift from lower linear region to upper linear region at faster rate, after being in the nonlinear state for some period. At $\gamma = 1$, depending on the variance of $y_t$ and size of $c$, discrimination between the nonlinear and linear series may not be significant. (Yaya and Shittu, 2011). The transition functions in (3) and (4) are bounded between 0 and 1, and this makes the STAR modelling of interesting application. When the transition function is at zero state, the entire system in (2) becomes linear, and at unity state, it is also linear. In most of the time, the transition function is such that $0 < F\left(y_{t-d}; \gamma, c\right) < 1$, which is a nonlinear state.

Specification between the asymmetric and symmetric transition function is often carried out using the approach outlined in Teräsvirta (1994). Though there is a newer specification approach proposed in Escribano and Jordà (2001), the approach of Teräsvirta (1994) is not dominated by that of Escribano and Jordà (2001). Further readings on the specification of STAR models are referred to the two articles as well as Luukkonen, Saikkonen and Teräsvirta (1988).
The Variance Equation: GARCH and ST-GARCH models

Apart from the issue of nonlinearity of the time series $y_t$, the innovations of the estimated model (mean equation) is often heteroscedastic for economic and finance series to be specific. Engle (1982) in his paper proposed the autoregressive Conditionally Heteroscedastic (ARCH) model of order $q$ for UK inflation.

$$\sigma^2_t = w + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 \quad (5)$$

where $\sigma^2_t$ is the conditional variance, $w$ is the constant and $\alpha_i \ (i = 1, \ldots, q)$ are the parameters in the ARCH model. The $\varepsilon_{t-i}$ are the residuals from the mean equation which are assumed to be heteroscedastic. Bollerslev (1986) proposed the generalized version of Engle’s model which is named the Generalized Autoregressive Conditionally Heteroscedastic (GARCH) model of order $(p, q)$ given as,

$$\sigma^2_t = w + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{r} \beta_j \sigma^2_{t-j} \quad (6)$$

where $\beta_j \ (j = 1, \ldots, r)$ are the parameters in the GARCH term. In the ARCH($q$) and GARCH($q, r$) models in (5) and (6), $w > 0$, $\alpha_i \geq 0$ and $\beta_j \geq 0$ and the existence of covariance-stationarity is $\sum_{i=1}^{q} \alpha_i < 1$ for ARCH($q$) and $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{r} \beta_j < 1$ for GARCH($q, r$) model. Hagerud (1996; 1997) and Gonzalez-Rivera (1998) considered introducing regime switching functional forms in the ARCH/GARCH systems. Their propositions are further developed in Lundbergh and Teräsvirta (1999). Hagerud (1996) proposed Smooth Transition-ARCH ($q$) (STARCH) model,

$$\sigma^2_t = w + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 \left[ 1 - F(\varepsilon_{t-i}) \right] + \sum_{i=1}^{q} \delta_i \varepsilon_{t-i}^2 F(\varepsilon_{t-i}) \quad (7)$$

where $w$ and $\alpha_i$ are as defined in ARCH model. The additional parameter, $\delta_i \ (i = 1, \ldots, q)$ defines the model in two-regimes. The transition function, with the transition variable $\varepsilon_{t-i}$ is of logistic and exponential as well. These are given as,

$$F(\varepsilon_{t-i}) = \frac{1}{1 + \exp(-\theta \varepsilon_{t-i})} \quad (8)$$

and

$$F(\varepsilon_{t-i}) = 1 - \exp(-\theta \varepsilon_{t-i}^2) \quad (9)$$

for the two forms respectively with $\theta > 0$ in both cases. The two transition functions in (8) and (9) will generate different data dynamics for the conditional variance. The logistic form in (8) will produce a return process where the dynamics of the conditional variance differ depending on the signs of the innovations (Hagerud, 1997). As $\varepsilon_{t-j} \to -\infty$, the logistic function equals $-1/2$ and as $\varepsilon_{t-j} \to +\infty$, the function equals $-1/2$. The exponential function in (9) is symmetric with respect to the sign of the error term, hence it generates data for which the dynamics of the conditional variance depends only on the magnitude of the innovations. As $|\varepsilon_{t-j}| \to \infty$, the impact of $\varepsilon_{t-i}^2$ on $\sigma^2_t$ changes smoothly from $\alpha_i$ to $\delta_i$, in both logistic ST-ARCH($q$) and ST-GARCH($q, r$) when the function equals 1, and as $\varepsilon_{t-j} = 0$, the logistic function equals 0. Also, as the parameter $\theta$ becomes larger, both the logistic ST-ARCH and ST-GARCH functions approach step functions which equal 0 for negative $\varepsilon_{t-i}$ and 1 for positive $\varepsilon_{t-i}$. Therefore, for logistic function, $-1/2 \leq F(\varepsilon_{t-i}) \leq 1/2$ and for exponential function, $0 \leq F(\varepsilon_{t-i}) \leq 1$. For positive conditional variance in the logistic ST-ARCH model, the condition $\alpha_i \geq \frac{1}{2} |\delta| \ 0 \ 0 \ \max(\delta, 0) \ \leq 1$. For the positive conditional mean in the exponential ST-ARCH, $\alpha_i + \delta \geq 0$ and for stationarity of the innovations $\varepsilon_{t-i}, \ \sum_{i=1}^{q} \max(\delta, 0) < 1 \ (Hagerud, 1997)$. The generalized form of the
model called Smooth Transition-GARCH \((q, r)\) (ST-GARCH) is proposed in Hagerud (1997) and Gonzalez-Rivera (1998) as,

\[
\sigma^2_t = w + \sum_{i=1}^{q} \alpha_i \varepsilon^2_{t-i} \left[1 - F\left(\varepsilon_{t-i}\right)\right] + \sum_{j=1}^{r} \delta_j \varepsilon^2_{t-j} F\left(\varepsilon_{t-j}\right) + \sum_{j=1}^{r} \beta_j \sigma^2_{t-j}
\]  

with the transition function in (8) and (9) for the logistic and exponential cases respectively. The ST-GARCH model only included the GARCH term, \(\sigma^2_{t-j}\). For positive conditional variance in the logistic ST-GARCH model, all the covariance stationarity condition of GARCH\((p, q)\) model hold here in ST-GARCH, and apart from these, \(\alpha_i \geq \frac{1}{2} |\delta_i|\) for the logistic case and for the stationarity of the innovations \(\varepsilon_t\), \(\sum_{i=1}^{q} \alpha_i - \frac{1}{2} |\delta_i| + \max(\delta_i, 0) + \sum_{j=1}^{r} \beta_j < 1\). For the positive conditional mean in the exponential ST-GARCH, \(\alpha_i + \delta_i \geq 0\) and for stationarity of the innovations \(\varepsilon_t\), \(\sum_{i=1}^{q} \alpha_i + \max(\delta_i, 0) < 1\) (Hagerud, 1997). A similar ST-GARCH \((p, q)\) is proposed in Anderson et al. (1999) and applied recently in Nam et al. (2002). This is given as,

\[
\sigma^2_t = \left( w_0 + \sum_{i=1}^{q} \alpha_i \varepsilon^2_{t-i} + \sum_{j=1}^{r} \beta_j \sigma^2_{t-j}\right) \left[1 - F\left(\varepsilon_{t-i}\right)\right] + \left( w_0 + \sum_{i=1}^{q} \alpha_i \varepsilon^2_{t-i} + \sum_{j=1}^{r} \beta_j \sigma^2_{t-j}\right) F\left(\varepsilon_{t-j}\right)
\]

This is a variant of GARCH model in regime switching functional form. The parameters and the conditions of existence of GARCH as defined for the GARCH specification in (6) holds for the ST-GARCH model. The model in (11) is defined only for the asymmetric function (8), and therefore, the ST-GARCH model is otherwise known as Asymmetric nonlinear Smooth Transition-GARCH (ANST-GARCH) model (Nam, et al., 2002). Asymmetries are important properties in returns series or innovations, \(\varepsilon_t\). Positive and negative innovations exert different reactions on the volatility (Mandelbrot, 1963) and therefore led to the propositions of different volatility models with different functional forms. Other asymmetric GARCH variants are the exponential GARCH (EGARCH) of Nelson (1991), Asymmetric Power ARCH (APARCH) model of Ding, et al. (1993) and Glosten Jagnetathan and Runkle (GJR-GARCH) model of Glosten, et al. (1993), with these as common ones. Franses and van Dijk (2003) is able to show that there is similarity between the ST-GARCH \((q, r)\) model of Hagerud (1997), even in the conditions of existence of conditional volatility and stationarity. Our selection of asymmetric variants of GARCH in this paper is based on similarity with STAR model and their abilities to realize smooth changing dynamics.

3. Monte Carlo Simulation Experiment

The DGP above is the mean equation. This is re-defined as,

\[y_t = 1.8 y_{t-1} - 1.06 y_{t-2} + (\phi_0 - 0.9 y_{t-1} + 0.795 y_{t-2}) F\left(s_{t}; \gamma, c\right) + \varepsilon_t\]

with \(\phi_0 = \{0, 0.2, 0.5\}\) and \(\varepsilon_t \sim N\left(0, 0.1 \sigma_t\right)\). The values of \(\phi_0\) are chosen such that the DGP will realized stationary series. At \(F\left(s_{t}; \gamma, c\right) = 0\), the resulting linear model has complex roots that are less than unity in absolute term, hence the process becomes nonstationary and there is possibility of explosion. At \(F\left(s_{t}; \gamma, c\right) = 1\), the behaviour of the process is influenced by the values of \(\phi_0\). For example, when \(\phi_0 = \{0, 0.2\}\), the resulting characteristic equation has complex roots that are less than unity in absolute terms, hence the system reverts back to stationary region. At \(\phi_0 = 0.5\), the roots of the characteristic equations are real and the system realize nonstationary series. In the asymmetric transition function, \(\gamma\) and \(c\) are fixed at \(\gamma = 100\) and \(c = 0.2\). The variance equations used in the simulations are the ARCH \((1)\), GARCH \((1,1)\), STARCH \((1)\), ST-GARCH \((1,1)\) and ANST-GARCH \((1,1)\). In the first two, \((w, \alpha, \beta) = \{0.02, 0.3, 0.6\}\), and in ST-ARCH \((1)\) and ST-GARCH \((1,1)\) models, \(\delta = 0.5\). In ANST-GARCH\((1,1)\), \((w_{10}, \alpha_{11}, \beta_{11}) = \{0.05, 0.5, 0.3\}\) and \((w_{20}, \alpha_{21}, \beta_{22}) = \{0.02, 0.3, 0.6\}\). The LSTAR and ESTAR functions for the innovations \(\varepsilon_t\) are \(F\left(\varepsilon_{t-1}\right) = \frac{1}{1 + \exp\left(-\theta \varepsilon_{t-1}\right)}\) and \(F\left(\varepsilon_{t-1}\right) = 1 - \exp\left(-\theta \varepsilon^2_{t-1}\right)\) respectively. The experiment is carried out over 1000 replications with sample sizes \(N = \{50, 100, 200, 500, 1000\}\). Initialization problem is catered for by discarding the first 100 observations in each replication. The relative frequencies of selecting an asymmetric STAR model with a particular variance equation are computed on every 1000 replications at 5% nominal significant level.
3.1 When the LSTAR DGP is used to realise LSTAR series.

Table 1: Selection Frequencies of models at different $\phi_{20}$ with fixed $\theta = 1$

<table>
<thead>
<tr>
<th>$\phi_{20}$</th>
<th>N</th>
<th>LSTAR-ARCH</th>
<th>LSTAR-GARCH</th>
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<td>TP</td>
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<td>0.5</td>
<td>50</td>
<td>0.522</td>
<td>0.934</td>
<td>0.692</td>
<td>0.879</td>
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<td></td>
<td>100</td>
<td>0.627</td>
<td>0.988</td>
<td>0.772</td>
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<td>200</td>
<td>0.707</td>
<td>0.999</td>
<td>0.903</td>
<td>0.996</td>
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<tr>
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$\phi_{20} = 0.2, \theta = 1$

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$\phi_{20} = 0.5, \theta = 1$

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Table 1 presents the results of the selections of LSTAR models with forms of heteroscedastic innovation processes. At zero intercept, $\phi_{20} = 0$ of the DGP, both LSTAR-GARCH and LSTAR-ANLSTGARCH models were detected at high frequencies better than the other model variants. As $\phi_{20}$ increased beyond 0, there was failure in model specifications as a result of matrix inversion problems encountered by the simulator. The results were worse when computed at the nonstationary region $\phi_{20} = \{0.2, 0.5\}$ of the DGP.

4. Conclusion

In this paper, we considered the specification of asymmetric Smooth Transition Autoregressive (STAR) models with linear and nonlinear GARCH innovations. The GARCH error specifications are those proposed already in the literature. Specifications of the Logistic STAR-GARCH (LSTAR-GARCH) variants were carried out using the usual STAR specification procedures. The empirical results showed strong support for modelling STAR models with different GARCH error specifications. The results further showed that STAR model in STAR-GARCH model cannot be misrepresented in the real sense.

References


