Probability of Default in Credit Risk Based on Discrete Trinomial Structure

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Abstract

Credit risk modeling has been the subject of considerable research interest in finance and has recently drawn the attention of statistical researchers. The distributions of defaults and transitions play the central role in the modeling, measuring, and managing of credit risk. Discrete Binomial Structure is one of the oldest approach to estimate default that defines a corporate bond in two states: default and no default at every period. This paper will introduce extended model for discrete binomial structure, namely Discrete Trinomial Structure. Discrete Trinomial Structure defines a corporate bond in three categories: investment grade, speculative grade, or default. We want to estimate probability of default of a corporate bond in the end of time maturity of the bond in each category. The result of this research is cumulative default probability from now until period *T* based on multiperiod default tree.

Keywords: mutiperiod default tree, rating category, reduced form model, transition matrix

1. Introduction

Credit risk is one of the most important financial risks in the markets. Credit risk is the risk induced from credit events such as credit rating change, restructuring, failure to pay, repudiation and bankruptcy. Thus whenever a contractual counterparty does not meet its obligations the creditor is subject to financial loss. Mathematical definition is given by Giesecke (2004), credit risk is the distribution of financial losses due to unexpected changes in the credit quality of a counterparty in a financial agreement.

Credit risk consists of two components, default risk and spread risk (Schmid, 2003). Default risk is the risk that a debtor will be unable or unwilling to make timely payments of interest or principal, i.e. that a debtor defaults on its contractual payments obligations, either partly or wholly. The default time is defined as the date of announcement of failure or deliver. Even if a counterparty does not default, the investor is still exposed to credit risk. Default probability is the probability that the debtor will default on its contractual obligations to repay its debt.

The oldest approach to estimate default is the historical method that focuses on counting historical defaults and rating transitions. Some newer statistical methods try to measure the probability that a debtor will be bankrupt in a certain period, given all information about the past default and transition behaviour and current market conditions.

2 Basics Setup

2.1. Rating Classes

A credit rating is an “evaluation of creditworthiness” issued by a rating agency. More technically, it has been defined by Moody’s, a ratings agency, as an “opinion of the future ability, legal obligation, and willingness of a bond issuer or other obligor to make full and timely payments on principal and interest due to investors.”

Public rating agencies such as Standard and Poor’s, Moody’s, and Fitch produce credit ratings for issuers of debt instruments. Rating agencies use proprietary models to classify the issuer and the bond issue into one of several discrete *credit rating classes*.

PT Pemeringkat Efek Indonesia (PEFINDO) is a credit rating agency in Indonesia. PEFINDO publishes annual "corporate default and rating transition study" to provide information on the consistency of PEFINDO‟s rating result with the default rate, which is often used as a proxy for probability of default.

Table 1 presents the interpretation of various credit ratings issued by the three major rating agencies, Moody's, Standard and Poor's, and PEFINDO. These ratings correspond to long-term debt; other ratings apply to short-term debt. Generally, the two agencies provide similar ratings for the same issuer.

Table 1. Long Term Obligation Ratings

S&P	**Moody**	**PEFINDO**	**Meaning**	**Categories**
AAA	C	idAAA	Highest quality, minimal credit risk	Investment Grade
AA	Aa	idAA	High quality	
A	A	idA	Strong payment capacity	
BBB	Baa	idBBB	Adequate protection, moderate credit risk	
BB	Ba	idBB	Likely to pay, but ongoing uncertainty	Speculative Grade
B	B	idB	High risk	
CCC	Caa	idCCC	Current vulnerability to default	
CC	Ca			
C	C		Nonpayment highly likely; In default (Moody's)	Default
D		idD	In Default	

Source: Carty & Lieberman, 1996; S&P CDO Surveilance, 2002; Budipratama, 2010

Ratings are broadly divided into:
1. Investment grade, that is, at and above BBB for S&P and Baa for Moody's
2. Speculative grade or below investment grade, that is BB, B, CCC and CC. This classification is sometimes used to define classes of investments allowable to some investors, such as pension funds.
3. Default. A rating default (C or D) as a failure of a company to pay any of its financial obligations in the form of either interest or principal on timely basis.

These ratings represent objective (or actuarial) probabilities of default.

2.2. Transition Matrices and Credit Migration

2.2.1 1-period transition.

To use agency ratings in a quantitative risk management model, we need to map the letter ratings to *credit migration* probabilities. In other words, we need to assign numbers to the likelihood that an issuer moves up or down in the credit class or even defaults. Rating agencies also provide this information, conveniently summarized in a *transition matrix*.

Each entry represents the probability of migrating from the row-class to the column-class. Note that each *row* adds up to 1. Also note that the large values in the diagonal reflect the "rating stability" aimed for by the rating agencies. We should also make the matrix square by adding one more row in the end for the "Default" state. This row would have zeros everywhere, except in the last entry which would be 100 (this assumes that default is an absorbing state) (Ross, 1996).

We can costruct a Grouped Transition Matrix for grouping rating at Table 2.

Table 2. Grouped Transition Matrix

Initial Category	Category at the year end (%)		
	Investment Grade (I)	**Speculative Grade (S)**	**Default (D)**
Investment Grade (I)	$p_{I,I}$	$p_{I,S}$	$p_{I,D}$
Speculative Grade (S)	$p_{S,I}$	$p_{S,S}$	$p_{S,D}$

2.2.2. N-period transition

Given the transition matrix, we can go even further and compute *n-period* transition matrices (probabilities of credit migration n-periods from now). For this, we need two assumptions:

1. *Time-invariance*. We assume that the 1-period transition matrix is constant, i.e., does not depend on calendar time.
2. *Markov-property*. Rating migration probabilities do not depend on anything else besides the current rating. In particular, history (upgrading/downgrading) is irrelevant.

To find the probability of going from state i to state j in 2 periods, one must consider all possible paths that lead from i to j. The probability is thus the sum of all such possible paths,

$$p_{ij}^{(2)} = \sum_{k=1}^{K} p_{ik} p_{kj}$$

where K = 8 for common transition matrix. Hence, the two-period transition matrix is

$$P^{(2)} = P \times P$$

In general, the n-period transition matrix is

$$P^{(n)} = P^n$$

2.3. Discrete Binomial Structure

Let $PD_i(t, T)$ denote the *probability of default* of obligor i from time t until time T. For instance, $PD_i(0, 1 \text{ year}) = 0.02\%$ means that there is a 0.0002 chance of default during the next year.

Consider a single obligor. To simplify the notation, let p_t denote the probability of default during period t, i.e., let $PD_i(t - 1, t) = p_t$. This is called the *conditional* default probability since it is the probability of defaulting at time t given that the firm survived until $t - 1$. It is also called the *marginal* default probability.

The term structure of default probabilities is the set of (cumulative or conditional) default probabilities 1-p_1 time S_1 ds. The possibility of default can be represented by a binomial tree. One period from now, the firm can be in one of two states: state of default (D_1) with probability p_1; or state of survival (S_1) with probability $1 - p_1$.

Note that there are many paths that lead to default, whereas only through one path does the firm remain alive. Denote by $PS(t, T)$ the probability of survival from t to T. The cumulative survival probability from now until period T is thus:

$$PD(0,1) = p_1$$

$$PS(0,1) = 1 - p_1$$

$$PS(0,2) = (1 - p_1)(1 - p_2)$$

$$PS(0,T) = (1 - p_1)(1 - p_2) \dots (1 - p_T) = \prod_{i=1}^{T} (1 - p_i)$$

The *cumulative default* probability until time T is the probability of defaulting at *any* point in time until time T. It is given by:

$$PD(0,T) = 1 - PS(0,T)$$

3 Discrete Trinomial Structure

This paper will introduce extended model for discrete binomial structure, namely discrete trinomial structure. Discrete Trinomial Structure defines a corporate bond in three categories: investment grade, speculative grade, or default (see Table 1). We want to estimate probability of default of a corporate bond in the end of time maturity of the bond.

In subsection 2.2 is said, there are two assumptions in n-period transition. These two assumption may be criticized. First, there is evidence that downgrading is more likely in recessions than in boom phases of the business cycle. Second, there is evidence that rating momentum matters, i.e., recently downgraded obligors are more likely to be downgraded again than other obligors who have been in the same rating class for a long time. Nonetheless, these two assumptions are used in practice since they allow considerable simplification of the credit risk models. So, this new model will estimate the probability of default with counting all transition probability from the beginning of life of the bond until the maturity date.

First, we see one period trinomial three n Figure 1. One assumptions we need to simplify the model. We use *p* as a notation for probability in the next period the bond will be in default category (*D*), *q* as a notation for probability in the next period the bond will be in speculative-grade category (*S*). So, we have *1-p-q* as a notation for probability in the next period the bond will be in investment-grade category (*I*)

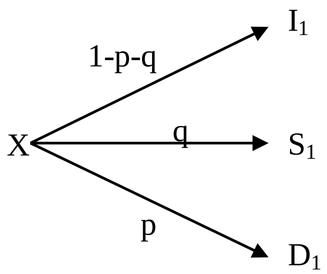

Figure 1. One Period Discrete Trinomial Structure

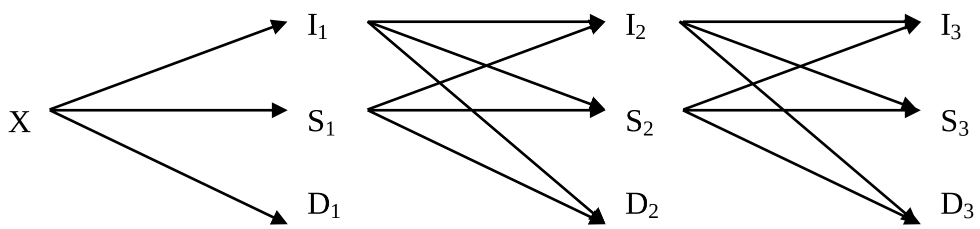

Figure 2. Multi Period Discrete Trinomial Structure

Denote by *PD*(*t*, *T*) the probability of default from *t* to *T*. The cumulative default probability from now until period *T* is thus:

$$PD(0,1) = p$$
$$PD(0,2) = p(1-p-q) + pq$$
$$\begin{aligned} PD(0,3) &= p(1-p-q)^2 + pq(1-p-q) + p(1-p-q)q + pq^2 \\ &= p(1-p-q)^2 + 2pq(1-p-q) + pq^2 \end{aligned}$$
$$\begin{aligned} PD(0,4) = {} & p(1-p-q)^3 + 3pq(1-p-q)^2 \\ & +3pq^2(1-p-q) + pq^3 \end{aligned}$$
$$\vdots$$
$$\begin{aligned} PD(0,T) = {} & \binom{T}{0} pq^0(1-p-q)^{T-1} + \binom{T}{1} pq^1(1-p-q)^{T-2} \\ & + \binom{T}{2} pq^2(1-p-q)^{T-3} + \cdots + \binom{T}{T} pq^T(1-p-q) \end{aligned}$$

$$= \sum_{i=0}^{T-1} \binom{T-1}{i} pq^i (1-p-q)^{(T-1)-i}$$

4 Example

For an example, we will compute cumulative default probabilities for a corporate bond until the maturity date. To begin, we use PEFINDO's transition matrix that is shown in Table 3.

Table 3. One Year Rating Transition Rate (1996-2010)

Initial Rating	Rating at the year end (%)								
	idAAA	**idAA**	**idA**	**idBBB**	**idBB**	**idB**	**idCCC**	**idD**	***NR***
idAAA	88.89	5.56	0.00	0.00	0.00	0.00	0.00	0.00	5.56
idAA	3.77	84.91	6.60	0.00	1.89	0.00	0.00	0.94	1.89
idA	0.26	8.88	82.77	2.09	0.78	0.00	0.00	3.39	1.83
idBBB	0.00	0.63	14.11	66.46	4.70	1.25	1.88	7.84	3.13
idBB	0.00	0.00	0.00	20.00	21.54	6.15	4.62	30.43	16.92
idB	0.00	0.00	0.00	8.70	13.04	34.78	4.35	30.43	8.70
idCCC	0.00	0.00	15.79	47.37	10.53	10.53	5.26	10.53	0.00

Source: Budipratama, 2010

The ratings in the first column are the starting or current ratings. The ratings in the first row are the ratings at the risk horizon. For example, the likelihoods in Table 4 corresponding to an initial rating of BBB are represented by the BBB row in the matrix. Further, note that each row of the matrix sums to 100%.

The information in Tables 3 is now used to costruct a Grouped Transition Matrix for grouping rating, as shown in Table 4.

Table 4. Grouped Transition Matrix

Initial Category	Category at the year end (%)		
	Investment Grade (I)	**Speculative Grade (S)**	**Default (D)**
Investment Grade (I)	91.23	2.63	3.04
Speculative Grade (S)	30.62	36.94	23.91

Consider a single BBB rated bond which matures in five years. So, the bond is in 'Investment Grade (I)' category. Let us first list possible credit outcomes that can occur at the end of the year due to credit events:

- the issuer stays at 'Investment Grade (I)' category at the end of the year;
- the issuer migrates down to 'Speculative Grade (S)' category; or
- the issuer defaults.

Each outcome above has a different likelihood or probability of occurring. We derive these from historical rating data and we assume that the probabilities are known. That is, for a bond starting out as BBB, we will compute the probability of default that this bond will defaults at the end of maturity. These cumulative probability of default is

$$PD(0,5) = \sum_{i=0}^{4} \binom{4}{i} pq^i (1-p-q)^{(T-1)-i}$$

$$= \binom{4}{0} pq^0 (1-p-q)^4 + \binom{4}{1} pq^1 (1-p-q)^3$$

$$+ \binom{4}{2} pq^2 (1-p-q)^2 + \binom{4}{3} pq^3 (1-p-q)^1$$

$$+ \binom{4}{4} pq^4 (1-p-q)^0$$

$$= p(1-p-q)^4 + 4pq(1-p-q)^3 + 6pq^2(1-p-q)^2$$

$$+4pq^3(1-p-q)^1 + pq^4$$

$$= (0.0304)(0.9123)^4 + 4(0.0304)(0.0263)(0.9123)^3$$
$$+6(0.0304)(0.0263)^2(0.9123)^2$$
$$+4(0.0304)(0.0263)^3(0.9123)^1 + (0.0304)(0.0263)^4$$
$$= 0.0235937$$

The result showed that issuers with credit rating of BBB (Investment Grade) have probability of default of 0.0236 by the end of the maturity date.

We can also compute probability of default for issuer with credit rating in "Speculative Category" until the maturity date with the same way. That is,

$$PD(0,5) = \sum_{i=0}^{4} \binom{4}{i} pq^i (1-p-q)^{(T-1)-i}$$
$$= \binom{4}{0} pq^0(1-p-q)^4 + \binom{4}{1} pq^1(1-p-q)^3$$
$$+ \binom{4}{2} pq^2(1-p-q)^2 + \binom{4}{3} pq^3(1-p-q)^1$$
$$+ \binom{4}{4} pq^4(1-p-q)^0$$
$$= p(1-p-q)^4 + 4pq(1-p-q)^3 + 6pq^2(1-p-q)^2$$
$$+4pq^3(1-p-q)^1 + pq^4$$
$$= (0.2391)(0.9123)^4 + 4(0.2391)(0.3694)(0.9123)^3$$
$$+6(0.2391)(0.3694)^2(0.9123)^2$$
$$+4(0.2391)(0.3694)^3(0.9123)^1 + (0.2391)(0.3694)^4$$
$$= 0.0498125$$

The result showed that issuers with credit rating in Investment Grade have probability of default of 0.0498 by the end of the maturity date.

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