Search for the best operating conditions of a soft drink bottling through optimum designs of experiments.

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Abstract
Taguchi´s off-line control methods place the quality improvement of products and processes at the design stages. Design of parameters plays a crucial role at these stages. This method is based on the recognition of two kinds of factors that may have impact on a quality characteristic: those that can be controlled at the design *-control factors-* and those that influence on the quality of the product when it is being used *-noise factors-*. The aim of this technique is to identify the optimum settings for the control factors that bring desired values for the quality characteristic with minimal sensitivity to noise factors. This methodology, named "Robust Parameter Design", has been widely spread but also criticized. Among the most important weaknesses, it can be mentioned that it involves experiments with a large number of runs, although the analysis only uses a summary of the observations not fully exploiting the collected information.
Optimum design of experiments theory provides a valid alternative to obtain the necessary information with a smaller number of runs than the traditional experimental designs strategies. There are three aspects to be considered before applying this method: the specification of a model to describe the relationship between the factors and the quality characteristic, the choice of an optimality criterion and the determination of the number of runs to be performed. The optimality criterion reflects the quality of the estimators of the model parameters. The objective of this strategy is to find the trials that must be run in order to estimate the parameters of the model so that the desired criterion is satisfied.
This work presents an overview of the fundamental aspects of optimum design of experiments theory and the innovative application of this technique to a soft drink bottling process. The objective of this study is to find the operating conditions that lead to the least loss of CO2 in the drinks between packaging and product purchase. The traditional designs are also presented for comparison purposes.

Key Words: Optimum Design of Experiments; Robust Parameter Design; Off-Line Control.

1. **Introduction.**

In the historical evolution of quality improvement strategies, three stages can be distinguished. The first emerged with the concept of inspection and its main activities were the detection and disposal or reprocessing of defective units. Subsequently, under the premise "quality is not inspected, quality is controlled", the second stage arose. At this stage, called on-line control, the control tasks were performed during the production processes. Statistical control processes played a key role in planning these activities. In the third stage, under the new premise "quality is not controlled, quality is designed" the off-line control appeared. Taguchi, the major exponent of these ideas, stated that the inspection and statistical control of processes were not sufficient to obtain competitive qualities. He suggested that it should be emphasized in the prevention by introducing quality at the phase of the products or processes design. The experimentation is the basic resource of the off-line control. The objective of this tool is to identify and quantify the effects that different parameters of the product or process have on relevant quality characteristics. The particularity of these experimental strategies is that it

+Unfortunately, she past way on last december.

distinguishes factors that can be controlled at the design phase -*control factors*- from those whose effects on the quality characteristic show up when the product is being used -*noise factors*-. This distinction, due to Taguchi, lead to the appearance of experimental plans for design of parameters intended to identify the optimum settings for the control factors that bring desired values of the quality characteristic of interest with minimum sensitivity to noise factors. Taguchi proposed experimental designs that consist of N_r treatments, specified by an experimental design for the noise factors, at each one of N_c treatments defined for the control factors. The data analysis is carried out summarizing the N_r values obtained from each one of the N_c treatments through a variable called Signal Noise Ratio (SNR). The SNR is chosen according to whether the interest is to minimize, maximize or achieve a target value of the quality characteristic.
This methodology has received several critics, mainly due to the fact that they do not make full exploitation of the data obtained from the experimentation. Consequently, new proposals appeared. Some of them suggest modeling the response behavior according to the values of the control and noise factors.
The approach of optimum design of experiments allows determining the trials that should be run so that the model parameters estimators meet an optimality criterion, being pre-specified the model to estimate and the number of trials to test. This approach offers an alternative methodology to estimate the parameters of interest of the model. Romero Zúnica (2002) and Romero et al. (2007) proposed the application of this methodology in robust parameter designs.

2. **Material and Methods.**

The experimental plans proposed by Taguchi, known as *crossed designs*, consist of a design with N_c treatments defined for the control factors -*inner array*-, and another one with N_r treatments for the noise factors - *outer array*. Each of the N_c treatments is run at each of the N_r conditions. In general, orthogonal arrays are used in both cases (*inner* and *outer arrays*) in order to obtain a number of trials not excessively large. The data analysis is carried out summarizing the N_r values at each N_c treatment using a measure called signal noise ratio (SNR). Then, the treatment that optimizes the SNR is sought by habitual procedures. Generally, the residual mean square of the ANOVA involved in the analysis has associated few degrees of freedom.
One of the alternatives proposed to make better exploitation of the collected information, is to model the response by:

$$E(Y) = \beta_0 + \sum_i \beta_i X_i + \sum_{i \neq j} \beta_{ij} X_i X_j + \sum_k \beta_k Z_k + \sum_{k \neq l} \beta_{kl} Z_k Z_l + \sum_i \sum_k \beta_{ik} X_i Z_k \quad i,j = \overline{1,c};\ k,l = \overline{1,r}$$

In this model, three groups of effects can be distinguished: the main effects and double interactions of the control factors; the main effects and double interactions of noise factors; and the double interactions between control and noise factors. If some of the effects in the last group are significant, the change in the response for the levels of the noise factors varies for different levels of the control factors.

The D-optimality criterion is widely used among several others (Atkinson ET to, 2007). A design is D-optimum, for a fixed number of trials, if it minimizes the value of the determinant of the variance and covariance matrix for the estimators of the model parameters. In geometric terms, an D-optimum design minimizes the volume of the confidence hyper-ellipsoid for the vector of parameters. Thus, this design is the one that provide the most precise estimators of the vector of parameters.
In some situations, the interest is to estimate s of the p model parameters as much precise as possible. In these cases, the adequate optimality criterion is D_S-optimality. This design is

defined similarly to the D-optimum one but over the determinant of the sub-matrix of the variance and covariance matrix corresponding to the s parameters of interest. To apply this criterion, the vector of parameters is partitioned in the s of interest and the $p - s$ remaining. In the case of robust parameters design, the interest would be to identify and accurately estimate the parameters of the first and third group.
An aspect to consider when applying this methodology is the evaluation of the goodness of the D- or D_S-optimum designs. The measures known as D- and D_S-efficiency compare the optimum designs found, called exact designs, with the absolute optimum design, which is the theoretical design that satisfy the desired optimality criterion but which is generally not possible to be run.
The seeking of exact designs is made through an iterative procedure that consists on proposing an initial design with a number of trials and improving it successively. There are several methods that carry out this procedure. One of them is Mitchell´s Detmax (Atkinson et al. 2007). In the case of experimental plans for parameters designs, as it was mentioned above, the particular interest may be in the terms associated to main effects and interactions of the control factors and interactions between control and noise factors.

In this study, a process of soda packaging is analyzed. Three control factors were considered: pressure with which CO2 is injected (*CO2*); needed torque to unscrew the lid (*tor*); and type of lid (*lid*). All these factors were run at two levels. The response variable is the CO2 content in the liquid. It was considered that the time that the soda remains in deposit once packaged (*tim*) and the temperature to which it stays (*tem*) could affect the response variable. These factors were also run at two levels. The performed experiment consisted of a complete factorial 2^5 design. In this work, we present the results of the comparison of the 2^5 design with a D_S −optimum design.

3. **Results**

As previously mentioned, the experiment was performed as a complete factorial design 2^5.The results obtained when analyzing such data are summarized in Table 1. The model considered at the initial stages of the study has 10 parameters and the response was expressed as:

$$E(Y) = \beta_0 + \beta_1 co2 + \beta_2 lid + \beta_3 tor + \beta_4 tem + \beta_5 tim + \beta_{14} co2 * tem + \beta_{15} co2 * tim + \beta_{23} lid * tor + \beta_{45} tem * tim$$

From Table 1, it is possible to see that all the effects included in the model were significant.

Considering that the model parameters can be estimated with less than 32 trials, it was considered an experimental plan with 16 trials. The estimation of main effects, double interactions between control factors, and double interactions between noise and control factors were of particular interest. A D_S-optimum design was sought, in which the parameter corresponding to the interaction between the noise factors was not important, although it could be significant. The set of candidate points was set up from the 32 treatments corresponding to the complete factorial design and Mitchell's Detmax algorithm was applied. The procedure was executed 100 times.

Table 1.- Estimations of the effects, model coefficients, standard errors and p-values according to the data collected in the complete factorial design.

Term	Effect	Coefficient	SE Coef.	T	p
Constant		0.45294	0.006814	66.47	<0.001
Co2	0.02512	0.01256	0.006814	1.84	0.079
Lid	-0.03775	-0.01887	0.006814	-2.77	0.011
Tor	-0.05287	-0.02644	0.006814	-3.88	0.001
Tem	0.24337	0.12169	0.006814	17.86	<0.001
Tim	0.26800	0.13400	0.006814	19.67	<0.001
Co2*Tem	0.10887	0.05444	0.006814	7.99	<0.001
Co2*Tim	0.09250	0.04625	0.006814	6.79	<0.001
Lid*Tor	-0.04550	-0.02275	0.006814	-3.34	0.003
Tem*Tim	0.10650	0.05325	0.006814	7.81	<0.001

In every 100 executions, the iterative procedure converged to a design. Six optimum experimental plans were found, having an efficiency of 100% with respect to the absolute D_S-optimum design.
The most remarkable result is that two of the experimental plans found as D_S-optimum correspond to the main and alternate factorial fractions 1⁄2. Therefore, these two plans are of resolution V and lead to orthogonal estimators of the effects of noise and control factors. The other plans are of resolution IV and also give answer to the problem since the effects are orthogonally estimated as well.
In Table 2 the estimated effects, coefficients and standard errors from data collected in the factorial fraction, one of the D_S-optimum designs, are presented.

Tabla 2.- Estimates of the effects, model coefficients, standard errors and p-values from data collected in the factorial fraction $1/2$.

Term	Effect	Coefficient	SE Coeff.	T	P
Constant		0.45225	0.008342	54.22	0.000
Co2	0.03550	0.01775	0.008342	2.13	0.077
Lid	-0.04925	-0.02463	0.008342	-2.95	0.026
Torq	-0.06500	-0.03250	0.008342	-3.90	0.008
Tem	0.25875	0.12937	0.008342	15.51	0.000
Tim	0.28050	0.14025	0.008342	16.81	0.000
Co2*Tem	0.11525	0.05762	0.008342	6.91	0.000
Co2*Tim	0.12000	0.06000	0.008342	7.19	0.000
Lid*Torq	-0.05225	-0.02613	0.008342	-3.13	0.020
Tem*Tim	0.11225	0.05613	0.008342	6.73	0.001

The comparison of the results presented in both tables allows seeing that the analysis of the data collected according to the factorial fraction design leads to the same conclusions that the analysis of the data collected according to a full factorial one. Furthermore, all effects were significant in both cases.
The loss in the precision of the coefficient estimations with the factorial fraction was small but it is interesting the cost reduction since the number of trials run in the experiment was reduced to the half.

4. **Conclusions.**

In this work, an application of optimum design of experiments was presented as an alternative to the experimental plans proposed by Taguchi in robust parameter designs problems. Optimum designs with half the number of trials than the original design, 16, were sought so as to estimate 10 parameters but with special interest in 9 of them. The data analysis showed that, in both cases, the same effects were significant. The optimality measures showed that the exact D_S-optimum designs that were found have an efficiency of 100% respect to the absolute D_S-optimum design.
It may be concluded that the theory of optimum designs of experiments is a useful way of reducing the number of trials involved in an experiment and so the costs of performing them, obtaining more economics designs than the traditional ones.
The application of the theory of optimum designs of experiments may be extended to situations in which quadratic effects and effects of another nature are considered. Other application field of optimum experimental designs that is being investigated is mixture designs. The advances show that this theory is also very benefical.

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