

Testing for Lack of Fit in Functional Regression Models

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We consider regression models with a response variable taking values in a Hilbert space, of finite or infinite dimension, and hybrid covariates. That means there are two sets of regressors, one of finite dimension and a second one functional with values in a Hilbert space. The problem we address is the test on the effect of the functional covariates. This problem occurs in many situations: testing the effect of the functional covariate in a semi-functional partial linear regression with scalar responses, significance test for functional regressors in nonparametric regression with hybrid covariates and scalar or functional responses, testing the effect of a functional covariate on a scalar or functional outcome. We propose a new test based on univariate kernel smoothing. The test statistic is asymptotically standard normal under the null hypothesis provided the smoothing parameter tends to zero at a suitable rate. The one-sided test is consistent against any fixed alternative and detects local alternatives a la Pitman approaching the null hypothesis at suitable rate. In particular we show that neither the dimension of the outcome nor the dimension of the functional covariates influences the theoretical power of the test against such local alternatives.

Key Words: Functional data, lack-of-fit test, regression, U-statistics