Asymptotic expansion methods for stochastic processes and their applications to statistics and finance

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Asymptotic expansion methods for semimartingales are today widely used in statistics and finance. Asymptotic expansion techniques for perturbed stochastic systems have been applied to option pricing as well as to the higher-order statistical inference. This methodology is based on the expansion of the Wiener or Wiener-Poisson functionals in the Sobolev spaces of the Malliavin calculus. On the other hand, as a refinement of limit theorems, we need a more distribution-theoretical expansion, and in fact, solutions are provided by the martingale approach and the mixing approach. The martingale expansion was first developed in the central limit case and applied to ergodic diffusions. This method was also applied to estimation of the linearly parameterized diffusion coefficient under high-frequent sampling with finite time horizon. However, the quasi-maximum likelihood estimator and the Bayesian type estimator of the volatility parameter are in general asymptotically mixed normal. Thus an expansion in the mixed normal limit is indispensable to develop the higher-order approximation and inference theory for the volatility. The classical methodology does not work because of the randomness of the characteristics of the limit distribution. In this talk, we discuss a new expansion method for a martingale having a mixed normal limit. The asymptotic expansion is described by an adjoint operation of a random symbol consisting of the adaptive random symbol and the anticipative random symbol. The former corresponds to the correction term appearing in the central limit case, and the latter is essentially new and described by the Malliavin calculus. We show applications to a quadratic form of a diffusion process ("realized volatility") and also to the power variation. Other applications are found in the quasi-likelihood analysis. The martingale expansion and the mixing expansion are regarded as complementary methods. Many applications of the mixing expansion to statistics for mixing Markovian processes are already known. Recently, it was applied to an estimation of the hidden structure of the volatility process.

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