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Hitting distributions of stable processes via path censoring and self-similarity

After Brownian motion, α-stable processes are often considered an exemplary family of processes for which many aspects of the general theory of Lévy proesses can be illustrated in closed form. First passage problems, which are relatively straightforward to handle in the case of Brownian motion, become much harder in the setting of a general Lévy process on account of the inclusion of jumps. A collection of articles through the 1960s and early 1970s, appealing largely to potential analytic methods for general Markov processes, were relatively successful in handling a number of first passage problems, in particular for symmetric α-stable processes in one or more dimensions.

However, following this cluster of activity, several decades have passed since new results concerning first passage identities for α-stable processes have appeared. The last few years have seen a number of new, explicit first passage identities for one-dimensional α-stable processes thanks to a better understanding of the intimate relationship between the aforesaid processes and positive self-similar Markov processes..

In this paper we return to the problem of Blumenthal, Getoor and Ray, published in 1961, which gave the law of the position of first entry of a symmetric α-stable process into the unit ball. Specifically, we are interested in establishing the same law, but now for a one dimensional α-stable process which enjoys two-sided jumps, and which is not necessarily symmetric. Our method is modern in the sense that we appeal to the relationship between α-stable processes and certain positive self-similar Markov processes. However there are two notable additional innovations. First, we make use of a type of path censoring. Second, we are able to describe in explicit analytical detail a non-trivial Wiener-Hopf factorisation of an auxiliary Lévy process from which the desired solution can be sourced. Moreover, as a consequence of this approach, we are able to deliver a number of additional, related identities in explicit form for α-stable processes.