The Structure of Space and

Long Term Behaviour of Fréchet Means

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In recent years, the statistical analysis of data of a non-Euclidean nature has become more frequent and important. One way to generalise the mean of Euclidean data to the wider context is to use Fréchet means which, on a metric space, are defined to be the points in the space where the appropriate Energy function achieves its global minimum. Fréchet means so defined have by now been widely used and many of their probabilistic and statistical features have been studied. For example, regarding their long term behaviour, the Strong Law of Large Numbers was obtained by Ziezold (1977); the Weak Law of Large Numbers was given by Kendall and Le (2011); the extrinsic Central Limit Theorem was established by Bhattacharya and Patrangenaru (2005); the intrinsic Central Limit Theorem was proved by Kendall and Le (2011), where the latter three are for data on Riemannian manifolds. All these results demonstrate that the global topological and geometrical structures of the underlying space both play an important role in the behaviour of Fréchet means. A good understanding of these relationships is crucial for developing appropriate statistical methods required, for example, for computation, estimation and inference. This talk will discuss some recent developments to illustrate further how the structure of spaces, both manifolds and non-manifolds, influences the long term behaviour of such Fréchet means.

Key Words: Central Limit Theorem; Laws of Large Numbers; statistical analysis of non-Euclidean data.