

Hitting Time Distributions for Denumerable Birth and Death Processes

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For an ergodic continuous-time birth and death process on the nonnegative integers, a well-known theorem states that the hitting time $T_{0,n}$ starting from state 0 to state n has the same distribution as the sum of n independent exponential random variables. Firstly, we generalize this theorem to an absorbing birth and death process (say, with state -1 absorbing) to derive the distribution of $T_{0,n}$. We then give explicit formulas for Laplace transforms of hitting times between any two states for an ergodic or absorbing birth and death process. Secondly, these results are all extended to birth and death processes on the nonnegative integers with ∞ an exit, entrance, or regular boundary. Finally, we apply these formulas to fastest strong stationary times for strongly ergodic birth and death processes.

Key Words: Birth and death process, eigenvalue, hitting time