Generalized Dyson Brownian motion, McKean-Vlasov equation and eigenvalues of random matrices

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Abstract

Using Itô's calculus and the mass optimal transportation theory, we study the generalized Dyson Brownian motion (GDBM) and the associated McKean-Vlasov evolution equation with an external potential. Under suitable condition on the external potential, we prove the existence and uniqueness of strong solution to SDE for GDBM. Standard argument shows that the family of the process of empirical measures of GDBM is tight and its accumulative points in the weak convergence topology are weak solutions of the associated McKean-Vlasov evolution equation, which can be realized as the gradient flow of the Voiculescu free entropy on the Wasserstein space over the real line. Under suitable condition on the external potential, we prove that the McKean-Vlasov equation has a unique solution and the empirical measure of GDBM weakly converges to this solution as the dimension of random matrix tends to infinity. Moreover, we study the longtime asymptotic behavior of the McKean-Vlasov equation for convex potentials. Finally, we discuss the double-well potentials and raise some conjectures. This is a joint work with Songzi Li and Yong-Xiao Xie.

Key words and phrases: Generalized Dyson Brownian motion, McKean-Vlasov equation, gradient flow, Voiculescu free entropy, Wasserstein distance.