Spatial Distributions in Banach spaces and related Depths and Quantiles

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Nowadays we are often faced with the challenge of analyzing data having dimensions much more than the sample sizes. Such data can be handled by viewing them as samples from infinite dimensional spaces (e.g., functional data generated from function spaces). Spatial distributions, depths and quantiles have been widely used for various nonparametric statistical procedures for data lying in finite dimensional spaces. We consider the notion of spatial distribution and the associated empirical process in certain Banach spaces. We discuss some geometric properties of the spatial distribution map along with Glivenko-Cantelli and Donsker type results for the empirical spatial distribution process. The notion of spatial depth using spatial distribution in smooth Banach spaces is discussed along with some applications using DD-plots. Spatial quantiles are introduced as suitable inverses of spatial distributions. We introduce a computationally feasible estimator of spatial quantile using finite dimensional approximation of the data. A Bahadur-type asymptotic linear representation and related weak convergence of this empirical quantile are presented. We also demonstrate the empirical spatial depths and quantiles using some real and simulated functional data.

Key Words: Bochner integral, Donsker property, Gateaux derivative, Glivenko-Cantelli property

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