

Symmetric Rearrangements Around Infinity with Applications to Levy Processes

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We prove a new rearrangement inequality for multiple integrals, which partly generalizes a result of Friedberg and Luttinger (1976) and can be interpreted as involving symmetric rearrangements of domains around infinity. As applications, we prove two comparison results for general Levy processes and their symmetric rearrangements. The first application concerns the survival probability of a point particle in a Poisson field of moving traps following independent Levy motions. We show that the survival probability can only increase if the point particle does not move, and the traps and the Levy motions are symmetrically rearranged. This essentially generalizes an isoperimetric inequality of Peres and Sousi (2011) for the Wiener sausage. In the second application, we show that the q -capacity of a Borel measurable set for a Levy process can only decrease if the set and the Levy process are symmetrically rearranged. This result generalizes an inequality obtained by Watanabe (1983) for symmetric Levy processes.

Key Words: Capacities, Rearrangement inequalities, Trapping problem, Wiener Sausage