Local Scoring Rules: A Versatile Tool for Inference

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In a number of applications, the specification of highly structured statistical models makes infeasible the computation of the likelihood function. In fact, computing the global form of the likelihood, and in particular finding the normalisation constant of the joint distribution, can be very demanding. One way proposed in the literature to sidestep this problem is to to adopt *composite likelihood* methods, such as the well-known pseudo-likelihood approach. Such methods are generally motivated by the issue of computational feasibility arising in the application of the likelihood method in high-dimensional data analysis. In this work we display composite likelihood as a special case of a general estimation technique based on *proper scoring rules*, which are methods for encouraging honest assessment of probability distributions. Just like maximum likelihood, such a rule supplies an unbiased estimating equation for any statistical model, and the theory of such equations can be applied to understand the properties of the associated estimator. When, as in the case of pseudo-likelihood, the rule has a simple *local* structure, this avoids the need to compute normalising constants.

In a Bayesian context, comparisons between two competing models are generally performed using the log Bayes factor, which can be interpreted as measuring by how much the predictive log score for one model is better then that for the other. However, Bayes factors are not well-defined when improper distributions are used for model parameters. We explore the possibility of replacing the log score by some other proper scoring rule as a yardstick for comparing the quality of statistical models. In particular we find that, for a certain class of such proper scoring rules, the problems with improper priors simply do not arise. Examples and applications will be given.

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