

## Variance Estimation in Nonparametric Regression

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We propose a new estimator for the error variance in a nonparametric regression model. We estimate the error variance as the intercept in a simple linear regression model with squared differences of paired observations as the dependent variable and squared distances between the paired covariates as the regressor. For the special case of a one-dimensional domain with equally-spaced design points, We show that the resulting variance estimator is not only asymptotically normal and root- $n$  consistent, but also reaches the optimal bound in terms of estimation variance. Our method can be easily extended to nonparametric regression models with multivariate functions defined on arbitrary subsets of normed spaces.

Key Words: Asymptotic normality, Difference-based estimator, Generalized least squares, Optimal bound, Residual variance