Depth and the Deepest Point in Infinite Dimensional Spaces

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A depth function is a statistical device for determining whether a point is close to the centre of a probability distribution or away from it. The concept of depth function was originally developed for probability distributions in finite dimensional Euclidean spaces. With the recent advancement of scientic techniques and measurement devices, we often come across data that have dimensions much larger than the sample sizes. Such data cannot be handled using standard finite dimensional multivariate techniques due to their high dimensionalities and low sample sizes. A common approach for handling such data is to embed them into suitable infinite dimensional spaces (e.g., functional data lying in function spaces) and model them using probability distributions in infinite dimensional spaces. There have been some recent attempts to extend the concept of depth function for probability distributions in Hilbert and Banach spaces. Some depth functions behave very differently in finite and infinite dimensional spaces while some others do not. The centre of a probability distribution, which can be defined as the deepest point, also has interesting properties in infinite dimensional spaces. There are important statistical consequences of such behavior of depth functions and their associated deepest points as will be demonstrated using data lying in infinite dimensional spaces. This is joint work with Anirvan Chakraborty.

Key Words: Band depth, fractional Brownian motions and bridges, half-region depth, half-space depth, integrated data depth, projection depth.