Fluid approximation and a time change for an evolution model

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In the Bare Bones Evolution Model of Sagitov, (see “Stochasticity in the adaptive dynamics of evolution: the bare bones”, J. Bio. Dynamics, 2011) the appearance of a new mutation in a resident established population is described as follows. The resident population, which is assumed to be around its carrying capacity K, evolves as a binary splitting process with probability of successful reproduction (division) dependent on the size of that population and also on the size of the new mutant population. The mutant population starts with a single individual and also evolves as a binary splitting with initially very high probabilities of successful division.

Approximating this system for high values of carrying capacity K leads to a dynamical system with a small noise, which initially starts near an unstable fixed point. By a classical approximation Theorem the population densities converge to that unstable fixed point on any fixed finite time interval. In other words, the evolution does not produce a sizable effect in finite time. The shortcoming of approximations of processes on finite time intervals has been discussed already in Barbour (1980). The effect of evolution is therefore observed on intervals increasing to infinity with the size of carrying capacity. The same effect of “wrong” approximation on finite intervals is equally observed in deterministic systems. In this work we show that one needs intervals that increase as log K and describe the fluid approximation in deterministic as well as stochastic settings. The limiting dynamics after the proposed time change is given by the same dynamics but different initial conditions. In this approximation population densities converge to a stable fixed point as time goes to infinity.

Key Words: Branching process, fluid approximation, bare bones evolution, carrying capacity, dynamical systems.