

Branching processes evolving in asynchronous environments

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A pure decomposable two-type branching process in an asynchronous random environment is considered under the quenched approach. We suppose that particles of this process produce offspring of their own type only and that the restriction of the evolution of the population to any of the two types leads to a single-type branching process evolving in random environment generated by a sequence of independent probability laws. Assuming that both processes are (individually) critical and that the logarithms of the mean number of offspring of different types are negatively correlated in each generation, we prove a Yaglom-type conditional limit theorem for the number of individuals in the process at a distant moment given survival of both types up to this moment and show that, contrary to the ordinary critical Galton-Watson processes the population sizes of both types are subject of asynchronous oscillations. The model under consideration may be treated as a predator-pray model in random environment where large mean number of children in the predator population in generation n leads to small mean number of children in the pray population in the next generation and vice versa, small mean number of children in the predator population in generation n leads to large mean number of children in the pray population in the next generation.

Key Words: Yaglom-type limit theorem, predator-pray model