Controlled branching processes: applications in Biology

Miguel González*, Cristina Gutiérrez, Rodrigo Martínez, Inés María del Puerto

Universidad de Extremadura, Badajoz, Spain. mvelasco@unex.es, cgutierrez@unex.es, rmartinez@unex.es, idelpuerto@unex.es

Branching processes are useful probabilistic models with which to describe population dynamics in the broadest sense. The notion of branching has been relevant in the development of theoretical approaches to problems in applied fields such as for instance growth and extinction of populations, Biology, Epidemiology, Cell proliferation kinetics, Genetics and algorithm and data structures. The branching model considered in the present work is the controlled branching process (CBP). This model is used to describe the evolution of populations in which a control of the population size at each generation is needed. This control consists of determining the number of individuals with reproductive capacity at each generation, mathematically through a random process. In practice, this branching model could describe reasonably the probabilistic evolution of populations in which, for various reasons of an environmental, social, or other nature, there is a mechanism that establishes the number of progenitors which take part in each generation. For example, in an ecological context, one can think of an invasive animal species that is widely recognized as a threat to native ecosystems, but there is disagreement about plans to eradicate it, i.e., while the presence of the species is appreciated by a part of the society, if its numbers are left uncontrolled it is known to be very harmful to native ecosystems. In such a case, it is better to control the population to keep it between admissible limits even though this might mean periods when animals have to be culled. Another practical situation that can be modeled by this kind of process is the evolution of an animal population that is threatened by the existence of predators. So that, in each generation the survival of each animal will be strongly affected by this factor, being necessary the introduction of a random mechanism to model the evolution of this kind of population. A CBP can be also used to model the evolution of the number of individuals of a population in which the rate of growth not only depends on the current population size but also on the distance between this size and the *carrying capacity* of the environment, that is the maximum population size that the environment can admit in view of its resources. In this work we review some of the main features of these processes (expected values, extinction conditions, long-term behavior) and develop new theory on particular CBPs of special interest in Biology.

Key words: controlled process, extinction, asymptotic behavior, carrying capacity.