Approximation with Multiscale Kernels

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Kernel-based approximation methods play an important role in modern statistical learning theory. They provide a unique flexibility in modelling and approximation of the most diverse problems.

The general idea behind kernel-based approximation methods is to choose a unique (problem dependent) kernel and to build the approximation to an unknown function from measured data using a finite linear combination of "shifts" of this kernel. The kernel is often translation-invariant or even radial. In many cases, the kernel depends on a scaling factor, which can be used to limit the influence of each single term within the approximation. Usually, this scaling factor is chosen uniformly, limiting the possibility to cope with different scales within the data.

In this talk, I will address the idea of using different shifts and scales of a compactly supported radial kernel to approximate data, so that the resulting approximation automatically reflects multiscales within the given data. I will present an efficient numerical scheme, which is based on residual-correction, and discuss its properties, including convergence, stability and computational complexity. If time permits, I will also discuss its relation and difference to other multiscale methods like wavelets.

Key Words: multiscale modelling, radial basis function, convergence, stability of numerical schemes