Small Area Estimation Applications in the US Census Bureau

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Abstract

We first discuss the problem of small area employment estimation that arises in the context of two important sample surveys of the U.S. Census Bureau ---- Annual Survey of Public Employment and Payroll (ASPEP) and Current Population Survey (CPS). The direct survey-weighted estimates of employment for small domains are highly variable. In this talk, we illustrate a small area estimation methodology to estimate employment by combining ASPEP with the previous census records using an empirical best prediction (EBP) methodology. The employment data are usually subject to skewness and heteroscedasticity and thus the well-known EBP methodology based on unit level linear mixed normal model does not fit well. In order to get around the problem, we apply a unit level linear mixed normal model on the log-transformed employment. We evaluate different competing estimates using the census data.

Keywords: Borrow Strength, EBLUP, Heteroscedasticity, Linear Mixed Model

1. Introduction

Over the last few decades, the U.S. Census Bureau has pioneered in developing innovative small area methodologies in different programs. In one of the most cited papers in small area estimation literature, Fay and Herriot (1979) developed a parametric empirical Bayes method to estimate per-capita income of small places with population less than 1000 and demonstrated, using the census data, that their method was superior to both direct design-based and synthetic methods. More recently, researchers at the U.S. Census Bureau implemented both empirical and hierarchical Bayes methodologies in the context of Small Area Income and Poverty Estimates (SAIPE) and Small Area Health Insurance Estimates (SAHIE) programs; see Bell et al. (2007) and Bauder et al. (2008).

Besides the Census Bureau's well-known SAIPE and SAHIE programs, researchers at the Government Division and the Current Population Survey (CPS) branch of the Demographic Statistical Division are actively pursuing state-of-the-art small area estimation techniques to improve on the current estimation methodologies for small areas. In particular, the CPS branch is in the process of extending and evaluating the well-known triple-goal estimation method, first proposed by Shen and Louis (1998), using the CPS data and administrative records. The triple-goal method is being pursued to meet the needs of multiple users interested in using estimates for different purposes, including ranking small areas in terms of a parameter of interest, identifying small areas with parameters above or below certain thresholds, and estimating parameter of individual small areas. Research findings from this project on multi-goal small area estimation will be reported in an upcoming ISI-IASS satellite meeting on small area estimation to be held in Bangkok, on September 1-4, 2013.

In this paper, we report our preliminary work on small area estimation for an important establishment survey involving government units. The Government Division of the U.S. Census Bureau conducts censuses of about 90,000 state and local government units every five years that have the year endings with 2 and 7 in order to collect data on the number of full-time and part-time state and local government employees and payroll. Between two consecutive censuses, the Government division also conducts the Annual Survey of Public Employment and Payroll (ASPEP), a nationwide sample survey covering all states and local governments in the United States, which include five types of governments: counties, cities, townships, special districts, and school districts. The first three types of government are referred to as general-purpose government, because they generally provide multiple government activities. Activities are coded as function codes. School districts cover only education functions. Special districts usually provide only one function, but can provide two or three functions. ASPEP is the only source of public employment data by program function and selected job category. Data on employment include the number of full-time and part-time employees, gross pay, and hours paid for part-time employees and are reported for the government's pay period covering March 12. Data collection begins in March and continues for about seven months. For more information on the survey, we refer to http://www.census.gov/govs/apes.

In 2009, ASPEP was redesigned and the old sample design was replaced by a systematic stratified probability proportional-to-size (PPS) cut-off sample design in order to reduce the sample size and respondent burden and at the same time to improve on the precision of the estimates and data quality. The sample design was implemented in multiple steps. First, a state-by-governmental type stratified PPS sample was selected, where size was taken as the total payroll (the sum of full-time pay and part-time pay from the employment portion of the 2007 Census of Government). In the second stage, a cut-off point was constructed to distinguish small and large government units in the stratum. Lastly, the strata with small-size government units were subsampled using a simple random sampling design.

The ASPEP survey is designed to produce reliable estimates of the number of full-time and part-time employees and payroll at the national level and for large domains (e.g., government functions such as elementary and secondary education, higher education, police protection, fire protection, financial administration, judicial and legal, etc., at the national level, and states aggregated by all function codes). However, it is also required to estimate the parameters for individual function codes within each state. This requirement prompted us to explore small area estimation methodology that borrows strength from previous census data as an alternative to collecting expensive additional data for small cells. We refer to Rao (2003) and Jiang and Lahiri (2006) for a comprehensive account of small area estimation theory and applications. In Section 2, we briefly describe our method. In Section 4, we present our findings from our data analysis.

2. Proposed Method

Let y_{ij} denote the number of full-time employees for the jth governmental unit within the ith small area ($i = 1, \cdots, m;\ j = 1, \cdots, N_i$). In this paper, we are interested in estimating the

total number of full-time employees for the i[th] small area given by $Y_i = \sum_{i=1}^{N_i} y_{ij}$ $(i = 1, \cdots, m)$. An estimator of Y_i is given by:

$$\hat{Y}_i = N_i \left[f_i \bar{y}_i + (1 - f_i) \hat{\bar{Y}}_{ir} \right], \quad (1)$$

where $\bar{y}_i = n_i^{-1} \sum_{j=1}^{n_i} y_{ij}$ is the sample mean; $f_i = n_i / N_i$, N_i and n_i are the sampling fraction, number of government units in the population and sample for area *i*, respectively; $\hat{\bar{Y}}_{ir}$ is a model-dependent predictor of the mean of the non-sampled part of area *i* $(i = 1, \cdots, m)$.

In this paper, we obtain $\hat{\bar{Y}}_{ir}$ using the following nested error regression model on the logarithm of the number of full-time employees at the government unit level:

$$\log(y_{ij}) = \beta_0 + \beta_1 \log(\bar{X}_i) + v_i + \varepsilon_{ij}, \quad (1)$$

$$v_i \overset{iid}{\sim} N(0, \tau^2) \text{ and } \varepsilon_{ij} \overset{iid}{\sim} N(0, \sigma^2), \quad (3)$$

where $\bar{X}_i$ is the average number of full-time employees for the i^{th} small area obtained from the previous census; β_0 and β_1 are unknown intercept and slope, respectively; v_i are small area specific random effects. The distribution of the random effects describes deviations of the area means from values $\beta_0 + \beta_1 \log(\bar{X}_i)$; ε_{ij} are errors in individual observations $(j = 1, ..., N_i;\ i = 1, ..., m)$. The random variables v_i and ε_{ij} are assumed to be mutually independent. We assume that sampling is non-informative for the distribution of measurements y_{ij} $(j = 1, ..., N_i;\ i = 1, ..., m)$. A similar model without logarithmic transformation can be found in Battese et al. (1988). The logarithmic transformation is taken to reduce the extent of heteroscedasticity in the employment data. Similar model using unit level auxiliary information was considered by Bellow and Lahiri (2012) in the context of estimating total hectare under corn for U.S. counties. We use the following model-based predictor of $\bar{Y}_{ir}$:

$$\hat{\bar{Y}}_{ir} \approx \exp\left[\hat{\beta}_0 + \hat{\beta}_1 \log(\bar{X}_i) + \hat{v}_i + \frac{1}{2} \hat{\sigma}^2 \right], \quad (4)$$

where $\hat{\beta}_0$, $\hat{\beta}_1$, $\hat{v}_i$, and $\hat{\sigma}^2$ are obtained by fitting (2) using PROC MIXED of SAS. We obtain our estimate of total number of full-time employees in area *i* using equations (1) and (4).

3. Data Analysis

For our data analysis, we first created a dataset by including only those government units that overlap between the 2002 and 2007 Census of Government units reporting strictly positive number of full time employees. The analysis covered 49 states, excluding Washington D.C, and Hawaii because we collected all the data in those two states.

We drew a sample from the 2007 Census of Government units and computed the following estimates of total full time employees for each of the 29 function codes available for all the local governments: direct Horvitz-Thompson estimate (denoted by HT), EBLUP estimate of Battese, Harter and Fuller (1988) (denoted by BHF), and our proposed estimate (log transformation). For a given function code, we compute: Percent Relative Error = 100(Estimate – True)/True (denoted by PRE), where true is the 2007 census full time employees for that function code. There are 1298 function codes across 49 states; only 241 of them (18.6 percent) show HT having better PRE where sample sizes are relatively large. Table 3.1 displays these percent relative errors for the three estimates for California only. From this table, it is clear that our proposed estimates are significantly better than the BHF estimates for all the function codes. Moreover, in 24 out of 29 function codes, our estimators are better than the HT estimators. We observe that our proposed estimates are generally better than both the methods for function codes with small sample size (n=2) like the Gas Supply.

Figures 3.1 and 3.2 display our residual analysis for our proposed model and BHF model for California. As you can see, the residual QQ-plot of our model is better than the BHF mode.

Figure 3.1: Residual Plot for California: Proposed Model

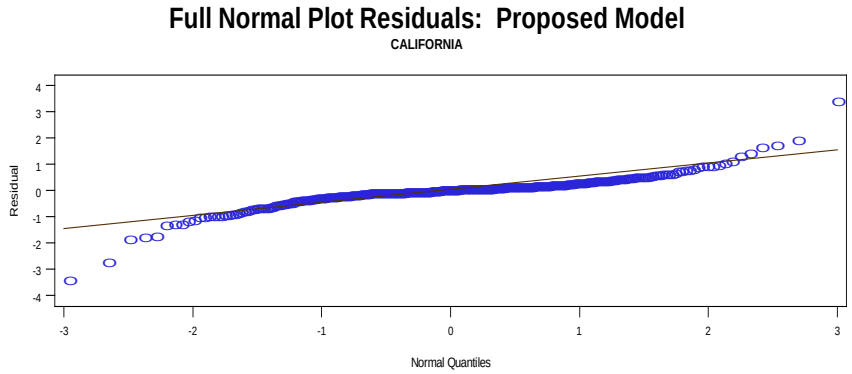
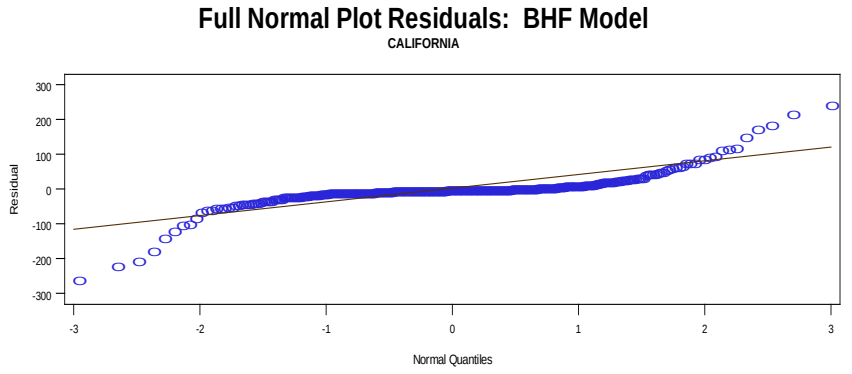


Figure 3.2: Residual Plot for California: BHF Model



We computed benchmarking ratios (BR) for both our model and BHF model. The BR is defined as $abs(\sum(est - HT)/\sum HT)$. The BR indicates how close the estimate to the HT

when considering at large areas. We defined a small size if the sample was smaller than 50 units. We estimated the BR for all the states by size. Table 3.2 summarizes the benchmarking ratios of the proposed model and the BHF model.

Table 3.1: Percent Relative Errors for Different Estimates of Full Time Employees- (California, in percentage)

Function	HT	Proposed	BHF
Airports	4.34	-0.49	-2.49
Correction	0.71	0.17	-3.46
Elementary and Secondary - Instruction	-1.52	-4.08	-27.7
Higher Education - Other	5.72	-0.19	-9.97
Higher Education - Instructional	4.48	0.86	-9.14
Financial Administration	-1.58	-0.65	-12.0
Firefighters	2.91	-1.36	-19.5
Judical & Legal	0.39	0.82	-2.21
Other Government Administration	-1.95	-0.12	-16.2
Health	-2.97	-0.08	-6.26
Hospitals	4.77	-0.71	-5.81
Streets & Highways	-3.36	0.11	-19.7
Housing & Community Development (Local)	-5.18	-2.11	-27.6
Local Libraries	5.72	-0.06	-10.6
Natural Resources	-3.74	-2.46	-25.0
Parks & Recreation	2.14	-2.11	-19.3
Police Protection - Officers	0.07	-0.21	-14.4
Welfare	-1.67	-0.14	-3.30
Sewerage	3.57	-1.91	-20.9
Solid Waste Management	3.73	-1.58	-12.3
Water Transport & Terminals	34.14	-1.64	-15.4
Other & Unallocable	-0.29	-1.65	-14.5
Water Supply	1.18	-7.20	-30.5
Electric Power	-1.28	-0.30	-4.87
Gas Supply	41.60	-11.8	-30.6
Transit	-1.37	-1.18	-8.49
Elementary and Secondary - Other Total	-0.87	-2.92	-22.6
Fire - Other	-9.03	-1.23	-10.1
Police-Other	2.04	-0.12	-11.3

Table 3.2: Comparison of Benchmarking Ratios

Size	BR of the proposed model	BR of the BHF model
< 50	1.5	1.7
> 50	1.1	1.4

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