Nonparametric Series Quantile Regression: Modeling, Estimation and Inference

Alexandre Belloni
Duke University, United States Alex Belloni abn5@duke.edu

Victor Chernozhukov
MIT, United States vchern@mit.edu

Ivan Fernandez-Val*
Boston University, United States ivanf@bu.edu

Quantile regression (QR) is a principal regression method for analyzing the impact of covariates on outcomes. The impact is described by the conditional quantile function and its functionals. In this paper we develop the nonparametric QR series framework, covering many regressors as a special case, for performing inference on the entire conditional quantile function and its linear functionals. In this framework, we approximate the entire conditional quantile function by a linear combination of series terms with quantile-specific coefficients and estimate the function-valued coefficients from the data. We develop large sample theory for the empirical QR coefficient process, namely we obtain uniform strong approximations to the empirical QR coefficient process by conditionally pivotal and Gaussian processes, as well as by gradient and weighted bootstrap processes. We apply these results to obtain estimation and inference methods for linear functionals of the conditional quantile function, such as the conditional quantile function itself, its partial derivatives, average partial derivatives, and conditional average partial derivatives. Specifically, we obtain uniform rates of convergence, large sample distributions, and inference methods based on strong pivotal and Gaussian approximations and on gradient and weighted bootstraps. All of the above results are for function-valued parameters, holding uniformly in both the quantile index and in the covariate value, and covering the pointwise case as a by-product. If the function of interest is monotone, we show how to use monotonization procedures to improve estimation and inference. We demonstrate the practical utility of these results with an empirical example, where we estimate the price elasticity function of the individual demand for gasoline, as indexed by the individual unobserved propensity for gasoline consumption.

Key Words: Quantile regression series processes, uniform inference, gasoline demand