Earthquake statistics and a FOSM seismic hazard analysis for a nuclear power plant in Taiwan

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Abstract

High seismicity is observed around Taiwan owing to the regional geological background. Because earthquake prediction is still not practical at this time being, seismic hazard analysis given earthquake observations is considered one of the options for earthquake hazard mitigation. This study first presents the statistics of major earthquakes in the past 110 years around the study site in North Taiwan, where a nuclear power plant is under construction. In addition, we introduce a new application of the First-Order-Second-Moment probability analysis to estimate the seismic hazard. The result shows that there is a 30% probability that PGA could exceed 0.3 g in 50 years, associated with a major earthquake with its magnitude greater than 6.0 occurring within 200 km from the study site. Such a best estimate in seismic hazard could be valuable to the earthquake-resistant design of the critical structure under construction.

Keywords: First-Order-Second-Moment, seismic hazard analysis, statistics of earthquakes in Taiwan

1. Introduction

The region around Taiwan is known for high seismicity because of the unique geological background. In average, there are around 2.5 major events (i.e., local magnitude greater than 6.0) per year since 1900 around the region. Among them, the M_w 7.6 (moment magnitude) Chi-Chi earthquake is the most infamous event causing severe casualty and economic loss in Central Taiwan in 1999.

Given the recent lessons such as the 2011 Japan Earthquake, earthquakes can be hardly predicted (Gellar et al., 1997; Mualchin, 2005; Wang et al., 2012a). Under the circumstance, alternatives such as seismic hazard analysis and earthquake early warning are considered practical methods for earthquake hazard mitigation (Geller et al., 1997; Wu and Kanamori, 2008; Wang et al., 2012b). In short, the difference between the two is that the former evaluates the earthquake probability before earthquakes occur, and the letter assesses their destructive power few seconds right after occurring.

The First-Order-Second-Moment (FOSM) is one of the common probabilistic analyses, evaluating both the mean and variance of the resulting random variable given those of input random variables. FOSM starts with the Taylor series expansion applied to the performance function $Y = g(X_1, X_2, \cdots X_n)$, and retains the first two terms up to the first order for approximating the mean and variance of Y (Hahn and Shapiro, 1967).

In this study, we introduce a new application of FOSM to seismic hazard analysis, evaluating the exceedance probability of earthquake-induced ground motion for a site in North Taiwan, where a nuclear power plant is under construction. The inputs for this analysis such as regional seismicity and ground motion models are also elaborated in this paper.

2. Seismicity around Taiwan since 1900

Fig. 1 shows the seismicity around Taiwan since 1900 (Wang et al., 2011; Wang et al., 2012c; Wang et al., 2013). Accordingly, there are more than 55,000 earthquakes occurring around the region in the past century. Fig. 2 shows the major events with local magnitude greater than 6.0 within 200 km from the study site in North Taiwan, where a nuclear power plant is under construction. Analyzing the 142 major events statistically, we presented the two histograms in Fig. 3 regarding earthquake size and source-to-site distance. It shows that the earthquake frequency generally decreases with the increase in earthquake magnitude, but for location or source-to-site distance, the distribution is more random with a higher concentration on 100 ~ 160 km. Table 1 summarizes the mean and variance of the two variables from this pool of observation.

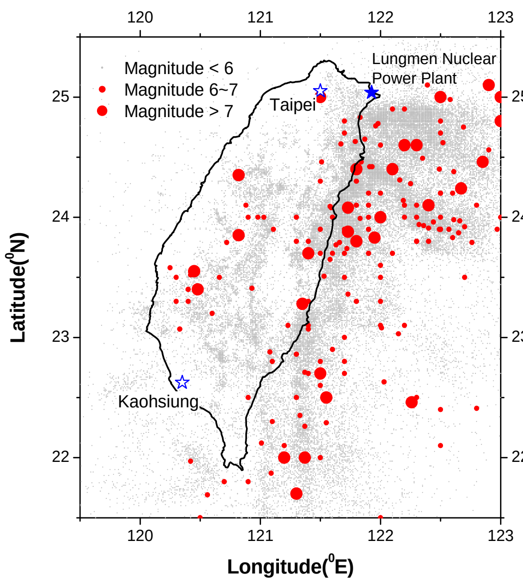

Fig. 1. The spatial distribution of more than 55,000 events since 1900

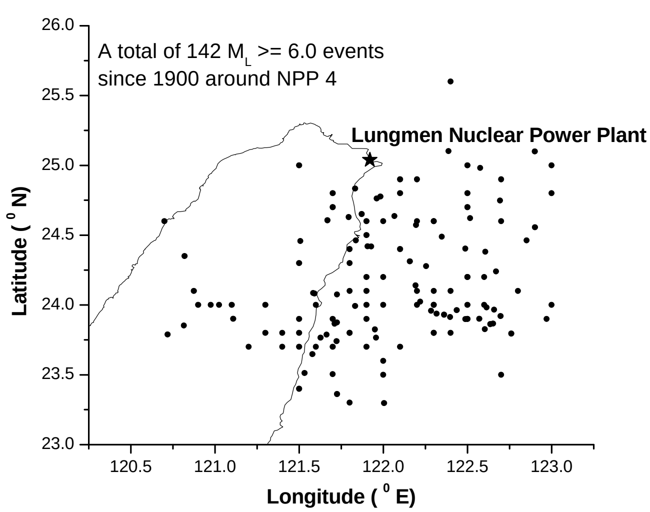

Fig. 2. The spatial distribution of 142 major events around the study site

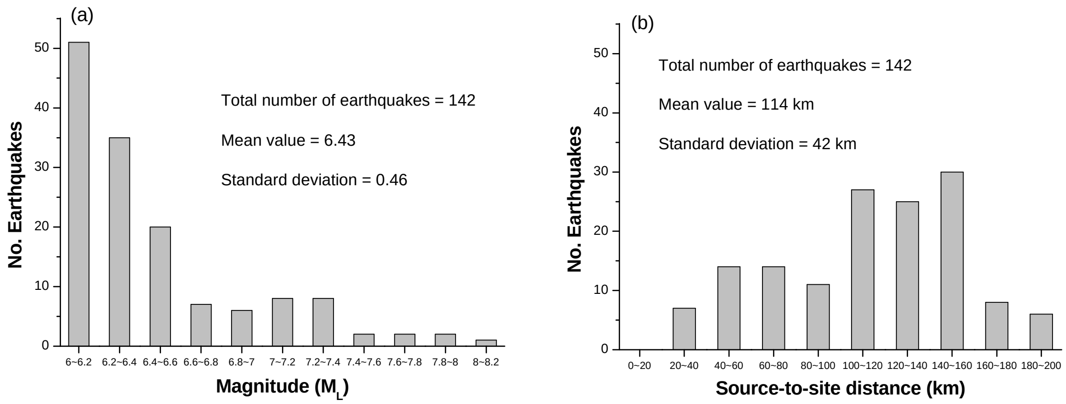

Fig. 3. Two histograms for the major event's size and location

Table 1. Statistics of the three input random variables and the output (i.e., lnPGA)

	Magnitude	Distance (km)	Model Error	lnPGA (g)
Mean	6.43	114	0	-3.93
Standard Deviation	0.46	42	0.57	1.14

3. Ground motion models

Ground motion models are a relationship developed with regression analysis to predict earthquake-induced ground motion levels at the site given the size and location of an earthquake. Cheng et al. (2007) analyzing the strong-motion data associated with local earthquakes occurring in Taiwan proposed the following empirical relationship to predict PGA (peak ground acceleration):

$$\ln PGA = -3.25 + 1.075M - 1.723\ln\left(D + 0.156\exp(0.624M)\right) + \varepsilon \qquad (1)$$

where M and D denotes the earthquake size (in M_w) and source-to-site distance (in km); ε is the model error owing to data scattering in the samples, or because of natural randomness. Based on the fundamentals of regression analysis, ε follows the normal distribution with its mean equal to zero, and its standard deviation can be computed from the scattered data. For this model, its standard deviation or the standard deviation of ε was found equal to 0.57.

4. FOSM-based seismic hazard analysis

Taking Eq. 1 as the governing equation of this seismic hazard analysis, we calculated the mean and variance of lnPGA with FOSM, given those of three random variables (summarized in Table 1) related to major earthquakes and the empirical model used. It is worth noting that the finite difference approximation developed by the U.S. Corps of Engineers (1997) was used during the FOSM calculation. The essence of this approach is given in the Appendix, along with the algorithms of FOSM.

The calculation shows that when a major earthquake occurs within 200 km from the site, the mean ($\mu_{\ln PGA}$) and standard deviation ($\sigma_{\ln PGA}$) of lnPGA at the site are -3.93 and 1.14, respectively. Also because lnPGA is considered following the normal distribution (Kramer, 1996; Cheng et al., 2007), the PGA exceedance probability against a given level y^* can be estimated as follows:

$$\Pr(PGA > y^*) = \Pr(\ln PGA > \ln y^*) = 1 - \Pr(\ln PGA \le \ln y^*)$$
$$= 1 - \Phi\left(\frac{\ln y^* - \mu_{\ln PGA}}{\sigma_{\ln PGA}}\right) \tag{2}$$

With the probability computation, Fig. 4 shows the PGA exceedance probability associated with a major event around the site. For example, there is a 0.5% probability that PGA could exceed 0.3 g when a major event with a random size and location occurs.

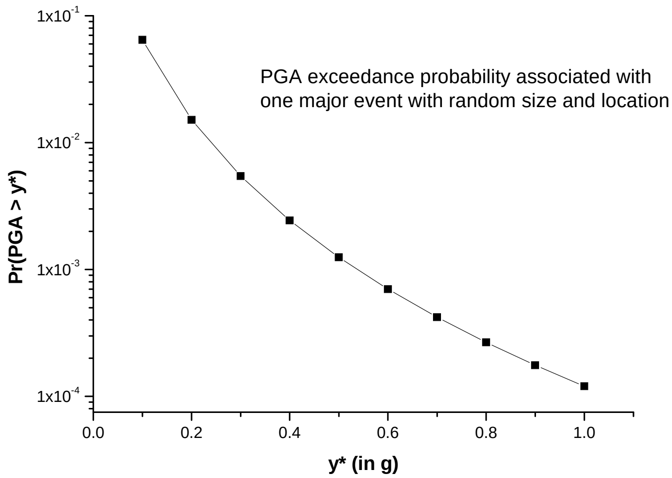

Fig. 4. PGA exceedance probability associated with a major event with random size and location

5. Annual rate of exceedance

Following the framework of Probabilistic Seismic Hazard Analysis (PSHA) (see Kramer, 1996), we estimate the annual rate of exceedance (λ) by combining the earthquake rate and the exceedance probability induced by one event, as follows:

$$\lambda_{PGA>y^*} = v \times \Pr(PGA > y^*) = v \times \left(1 - \Phi\left(\frac{\ln y^* - \mu_{\ln PGA}}{\sigma_{\ln PGA}}\right)\right) \tag{3}$$

where v is the annual rate of earthquakes, which is 1.4 per year in this study. Therefore, Fig. 5 shows the annual rate of PGA exceedance at the site in North Taiwan. For example, the rate of PGA exceeding 0.3 g is 0.007 per year, associated with the recurrence of major earthquakes.

Following another common application of the Poisson model to rare events (Kramer, 1996; Wang et al., 2012d), its recurrence probability within a given time interval t^* governed by the exponential distribution can be estimated as follows:

$$\Pr(T \le t^*; \lambda) = 1 - e^{-\lambda t^*} \tag{4}$$

As a result, Fig. 5 also shows the PGA exceedance probabilities in 25 and 50 years. Accordingly, there is a 30% probability that PGA can exceed 0.3 g in 50 years caused by a major event occurring within 200 km from the site.

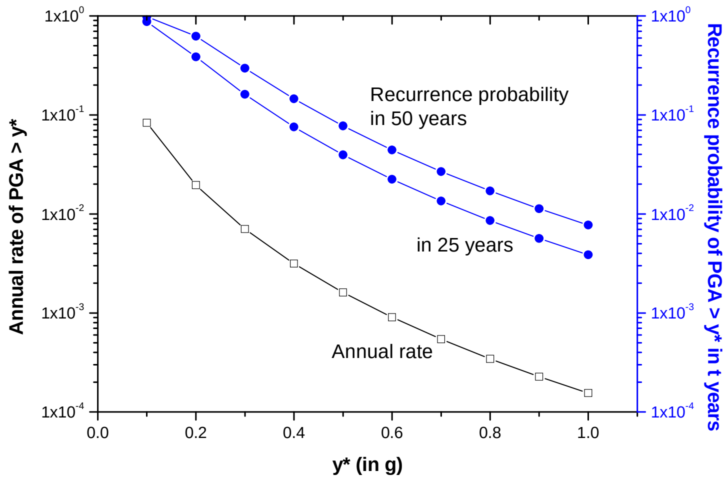

Fig. 5. Annual rate of PGA exceedance and its recurrence probability in 25 and 50 years

6. Conclusions

From the intense seismicity record around Taiwan since 1900, this study first estimated the mean and variance of the major event's size and location around the study site, where a nuclear power plant is under construction in North Taiwan. With an empirical model, the earthquake-induced ground motion becomes a function of three random variables: earthquake size, location, and the model's uncertainty. We then used FOSM to solve the governing equation on a probabilistic basis to evaluate the mean and variance of lnPGA, which is a new seismic hazard analysis different from existing approaches.

Given the statistics of size and location from 142 major events in the past 110 years, the PGA exceedance probability, annual rate of PGA exceedance, and PGA recurrence probabilities in 25 and 50 years are given in this paper. For example, there is a 30% probability that PGA can exceed 0.3 g in 50 years at the study site in North Taiwan. This best estimate should be valuable to the design of the critical structure that is being constructed at the site.

Appendix: the algorithm of FOSM

Given a function of random variables $Y = g\left(X_1, X_2, \cdots X_n\right)$, or referred to as a performance function, FOSM calculated the mean and variance of Y as follows:

$$E(Y) \approx g\left(E(X_1), E(X_1), \ldots, E(X_n)\right) \tag{A.1}$$

$$V(Y) \approx \sum_{i=1}^{n} \left[\left(\frac{\partial Y}{\partial X_i} \right)^2 V(X_i) \right] + 2 \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\frac{\partial Y}{\partial X_i} \frac{\partial Y}{\partial X_j} Cov(X_i, X_j) \right) \quad ; \quad for \;\; i < j \tag{A.2}$$

where $E(X)$ and $V(X)$ denote the mean and variance of a random variable X; Cov$[X_i, X_j]$ denotes the covariance between X_i and X_j. When any of two input variables is independent of each other, Eq. A.2 can be rewritten as follows:

$$V(Y) \approx \sum_{i=1}^{n} \left[\left(\frac{\partial Y}{\partial X_i} \right)^2 V(X_i) \right] \tag{A.3}$$

In Eq. A.3, the derivatives can be approximated with the following algorithm (U.S. Corps of Engineers, 1997). Take $\frac{\partial Y}{\partial X_1}$ for example, it can be calculated as follows:

$$\frac{\partial Y}{\partial X_1} = \frac{g(\mu_1 + \sigma_1, \mu_2, ..., \mu_n) - g(\mu_1 - \sigma_1, \mu_2, ..., \mu_n)}{2\sigma_1} \tag{A.4}$$

where μ_1 and σ_1 denote the mean and standard deviation of X_1.

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