Testing Spatial Clustering Using Relative Density of Two Random Geometric Digraph Families

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Abstract

We compare the relative density of two parameterized random geometric digraph families called proportional edge and central similarity *proximity catch digraphs* (PCDs). These PCD families were defined based on relative positions of data points from two classes and the relative density of these PCDs were previously used as a statistic for testing segregation and association patterns against complete spatial randomness. In this article, we extend the distribution of the relative density of central similarity PCDs for expansion parameter being larger than one, and compare finite sample performance of the tests by Monte Carlo simulations and asymptotic performance by Pitman asymptotic efficiency. We find the optimal expansion parameters of the PCDs for testing each alternative in finite samples and in the limit as the sample size tends to infinity. As a result of our comparison, we demonstrate that in terms of empirical power (i.e., for finite samples) relative density of central similarity PCD has better performance (which occurs for expansion parameter values larger than one) under the segregation alternative, while relative density of proportional edge PCD has better performance under the association alternative. The methods are also illustrated in a real-life data set from plant ecology.

Keywords: association, complete spatial randomness, consistency, Delaunay triangulation, Pitman asymptotic efficiency, proximity catch digraphs, segregation, U-statistic

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