

A Comparison of Risk Return Relationship in the Portfolio Selection Models

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The foundation of modern investment theory is laid upon the quadratic program portfolio selection model developed more than half century ago by Harry Markowitz. The optimization (risk-minimization) process over mean-variance-covariance space can trace out the efficient frontier curve, which provides the solution space for investors. However, an exact solution cannot be found without the knowledge of a risk free rate on a government bond and an investor's attitude toward risk. To this end, William Sharpe formulated and solved the angle-maximization model in which the risk (standard deviation) adjusted portfolio return (net of risk free rate) is maximized. The Sharpe model provides a convex combination of risk free government bonds and a portfolio of stocks selected based on the criterion of risk minimization. Does the choice of mean returns of stocks in the portfolio matter in the selection process? If so, how different are the optimum solution sets? In this note, we first apply the well-known means: arithmetic, geometric and harmonic means to five companies stocks. In addition, we add a golden mean to the simulation for comparison. We calculate four different kinds of means: arithmetic mean (AM), geometric mean (GM), harmonic mean (HM), and golden mean (GDM) - to investigate the risk-return contour using Markowitz risk minimization and Sharpe's angle maximization models. For a given k value (target portfolio return), the rank order of risk or variance-covariance (v) can change. In the vertical segment of an efficient frontier curve, we observed $v(\text{GDM}) > v(\text{HM}) > v(\text{GM}) > v(\text{AM})$. At higher k values, the rank changes to $v(\text{GDM}) > v(\text{HM}) > v(\text{AM}) > v(\text{GM})$. That is to say, ranking a portfolio using different kinds of means may well give different rankings depending on what k value one is evaluating. It is also shown the harmonic mean should not be used in the case of a small negative growth rate in stock prices.

Key Words: Markowitz risk minimization, Sharpe's angle maximization, different kinds of means, different rankings