Anderson-Darling type goodness-of-fit statistic based on a multifold integrated empirical distribution function

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An Anderson-Darling type goodness-of-fit statistic constructed from multifold integrated empirical distribution function is proposed. The proposed statistic is of an integral form whose integrand is a standardized square of m-fold integrated empirical distribution function. The empirical distribution functions is adjusted in advance so that it does not contain the components of m-th or lower degree polynomials. The proposed statistic is a natural extension of the goodness-of-fit statistic by Anderson and Darling (1952, *AMS*), which corresponds to the case $m = 0$. When $m = 1$, the proposed statistic has much statistical power to detect the discrepancy of dispersion of distribution. The Karhunen-Loève expansion of the limiting integrand process is obtained with Legendre eigenfunctions, and the limiting distribution of the proposed statistic is proved to be a weighted sum of chi-square random variables with the weights $1/\{k(k+1)\cdots(k+2m+1)\}$, $k = 1, 2, \ldots$. The explicit form of the Laplace transform of the limiting distribution without infinite product is derived. The relationship to a boundary-value problem is pointed out. Finally, it is mentioned that the similar type of extension is possible to Watson's (1961, *Biometrika*) statistic for testing uniformity of directional data.

Keywords: Boundary-value problem, Directional data, Karhunen-Lòeve expansion, Laplace transform, Legendre polynomial.