

# A Projection Based Nonparametric Test of Conditional Quantile Independence with an Application to Banks' Systemic Risk

Milan Nedeljkovic\*

Singidunum University, Belgrade, Serbia, [mnedeljkovic@fefa.edu.rs](mailto:mnedeljkovic@fefa.edu.rs)

This paper proposes a nonparametric (kernel) procedure for testing conditional quantile independence using projections. The interest in testing for conditional dependences arises in different fields of science. There are numerous examples in empirical finance and economics when one may be interested in testing conditional independence, either as a testable implication of an economic model or an empirical fact that can be exploited in forming trading strategies. A similar interest arises in neuroscience where causal interpretations (Granger causality) can be investigated by looking at the conditional dependence relations. The conditional quantile framework provides a general and flexible tool for investigating the conditional dependence structure of the variables of interest. We extend the existing nonparametric literature on testing conditional quantile dependence in several directions. First, we propose a method to reduce the curse of dimensionality by replacing a nonparametric alternative with a class of single-index semi-parametric alternatives. The proposed test is based on correlations between the residuals from the null model and the particular class of functions of linear combinations of the conditioning variables. By considering all linear combinations (with weights on the unit sphere) of the conditioning variables the test remains consistent against all deviations from the null hypothesis. Relative to the existing literature it can detect the high frequency local alternatives that converge to the null hypothesis in probability at faster rate while also offering improvements in the finite sample power when more variables are included under the alternative. In addition, it allows the researcher to include qualitative information if desired and direct the test against specific subsets of alternatives without imposing any functional form on them. Second, we use the weighted Nadaraya-Watson (WNW) estimator of the conditional quantile function avoiding boundary problems in estimation and testing and prove weak uniform consistency (with rate) of the WNW estimator for absolutely regular processes. The procedure is applied to a study of risk spillovers between the commercial banks. We show that the methodology generalizes some of the recently proposed measures of systemic risk and we use quantile framework to propose an alternative measure of systemic risk and the importance of individual banks.

**Keywords:** Conditional quantiles; U-statistic; Stochastic process; Systemic banks