Frequency Analysis of Heavy-Tailed Phenomena

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Heavy-tailed phenomena have been observed in various fields such as telecommunication networks (Internet), insurance, finance, seismology, to name a few. It is typical for these phenomena that they are rare and serially dependent. The tools of classical time series analysis (autocorrelation function, spectral density) are not suitable for describing extreme events and their dependence structure. However, in our approach we adapt the autocorrelation function from the time domain and the spectral density from the corresponding frequency domain of time series analysis: we apply them to indicator functions of the rare events of interest. Thus we deal with stationary sequences of indicator functions whose distributions change in dependence on a sufficiently high threshold. Therefore the classical results of time series analysis are not directly applicable: one deals with a triangular array of stationary sequences whose marginal distribution changes with the threshold. First, we introduce the *extremogram* as analog of the autocorrelation function of a stationary sequence and its sample version based on counts of rare events in a stationary sequence. Second, we define a corresponding periodogram, called *ex-periodogram* as an estimator of the spectral density defined via the extremogram. The ex-periodogram shares various of the classical properties of the periodogram of a stationary weakly dependent sequence: the ex-periodogram ordinates at distinct (Fourier) frequencies are asymptotically independent and exponentially distributed and smoothed versions are consistent estimators of the spectral density. Having established some basic asymptotic theory for the ex-periodogram our next goal is to apply the theory for estimating the spectral distribution function, parameter estimation of suitable time series models and goodness of fit tests. In this context, we study different versions of the integrated ex-periodogram. It is our objective to derive results parallel to the classical theory for the integrated periodogram of a stationary sequence which can be interpreted as a spectral empirical distribution. We show that the integrated ex-periodogram satisfies a functional central limit theorem under mild conditions, which ensure that goodness-of-fit tests, such as the Grenander and Rosenblatt tests and the Cramer-von Mises tests, can be constructed for the ex-periodogram. This means that we can determine whether a model is suitable solely based on the behavior of extreme event. Both simulated data and real-life financial data are used to illustrate the application of the above results.

Key Words: Heavy-tailed phenomena, frequency analysis, extreme value theory, time series analysis.