An International Comparative Study on the Degree of Difficulty of Primary School Mathematics Textbooks

Kuang Kongxiu, Yang Xinrong ,Cai Qingyou, Song Naiqing(corresponding author)
School of Mathematics and Statistics, Southwest University, Chongqing, China

Abstract

The understanding of the degree of difficulty of primary or secondary school textbooks internationally is significant for a country to develop its own primary or secondary school textbooks. This study first constructed a mathematical model to examine the degree of difficulty of primary school mathematics textbooks based on literature review, interviews with mathematics educators, and followed up exploratory factor analysis. Then the degree of difficulty of 12 sets of the most popularly used and representative primary school mathematics textbooks in 10 countries, including China, Japan, South Korea, Singapore, Russia, the United States, the United Kingdom, Germany, France and Australia was analyzed with the use of the mathematical model. Results suggest that the degree of difficulty of primary school mathematics textbooks used in these countries can be sorted from high to low as the following order: France(Belin Publishers), Russia (The 21st Century Press), Germany(Schroedel), Germany(Klett), China(People's Education Press), Japan (啓林館), South Korea (Doosan Press), Australia(Thomson Nelson), Singapore (Marshall Cavendish Education Publishers), Australia(Pearson Companies), The United States(Pearson Companies), British (Collins Press).

Key Words: Primary school mathematics; textbooks; model on difficulty; international comparison

1. Introduction

The understanding of the degree of difficulty of primary or secondary school textbooks internationally is significant for a country to develop its own primary or secondary school textbooks. For example, the burden of mathematics learning of Chinese primary school students has long been criticized by academic researchers and the public as being heavy. Although reasons contributing to this might be complicated, it is reasonable to conjecture that a main factor might be the reason that Chinese primary school mathematics textbooks are too difficult? If it is really so, to the reduction of the difficulty of Chinese school mathematics textbooks might be an effective means to reduce the burden of students´ learning? In view of this, this exploratory study tried to develop a mathematical model to measure the degree of difficulty of primary school mathematics textbooks. With the use of this model, 12 sets of the most popularly used and representative primary school mathematics textbooks in 10 countries were chosen for this comparative study.

2. Literature review

Since the 1980s, with the importance attached to mathematics education and the worldwide mathematics curriculum reform at primary and secondary school level, particularly, after the carry out of large international comparative studies in mathematics achievements, such as TIMSS and PISA, there has been an ever increasing research interest to investigate mathematics textbooks internationally. However, in literature, so far, every few studies have tried to investigate the degree of difficulty of primary and secondary school mathematics textbooks.

Among the very few studies focusing on the analysis and comparing the difficulty of textbooks, Conard Sue Stankewitz（1981）surveyed the views of 303 elementary school educators and 98 textbook publishers regarding the suitable reading levels of textbooks used for instruction in reading and content area subjects, examined preferences for difficulty of reading textbooks considering students' achievement levels and for reading content area textbooks when selections are made for individual students, groups, and classes. Furthermore, they examined techniques and testing methods used by educators and publishers to determine suitable reading levels.

Davison Alice (1986) suggested a view that well-organized, appropriately written, the use of ordinary vocabulary, and free use of sentence constructions could facilitate the reading of textbooks. They further discussed various ways that might lead to the difficulty of textbooks and proposed what and how to look for the factors which determine the degree of difficulty textbook texts.

Chall et al. (1991) proposed that the readability of the text would affect the degree of difficulty of textbooks. The readability of the text could be estimated by the intrinsic characteristics of the text, such as the frequency of obscure words, the difficulty of content or concepts, the complexity of sentence structures, organization and consistency. Basing on this, they constructed a widely used model to measure the readability of textbooks. Similarly, from the perspective of text linguistics, such as syntactic complexity, readability, Maricela Corzo-Pena (1996) analyzed the difficulty of primary school science textbooks for grades three and four and explored its relationship with the upper primary school students' reading comprehension.

In mathematics education research field, Nohara (2001) first proposed the overall difficulty of mathematical problem in a report submitted to the U.S. National Center for Education Statistics, which includes four factors: 1)the percentage of "scalability issues" (the so-called scalability problem refers to problems that require students draw their own conclusions and explain the process of problem-solving); 2)the percentage of problems with "real background"; 3)the percentage of problems with "operation" excluding the problems in the area of "amount"; 4) the percentage of problems of "multi-step reasoning".

Bao (2002) suggested that factors such as background, calculating, amount of knowledge, reasoning, and exploring would jointly influence the difficulty of a mathematics problem. Basing on these five factors, he developed a model, $d_i = \frac{\sum_j n_{ij} d_{ij}}{n} (\sum_j n_{ij} = n; i = 1,2,3,4,5; j = 1,2,...)$ to measure the comprehensive difficulty of a mathematics problem. In this model, d_i (i = 1,2,3,4,5) refers the value of "background", "calculating ", "reasoning", "amount of knowledge ", and "exploring" respectively; d_{ij} is the weight of the j-th level of the factor d_i ,which is taken by 1, 2, ... for its level; n_{ij} is the total of the j-th level of the factor d_i, its sum is equal to the total of the group of questions(n). Basing on this model, Bao further compared the difficulty of intended mathematics curriculum, implemented mathematics curriculum, and enacted mathematics curriculum in China and the United Kingdom.

Shi et al (2005) argued that the degree of difficulty of curriculum is influenced at least by three basic elements: the depth of curriculum, the width of curriculum, and time. The depth of curriculum refers to the depth of thinking required by the curriculum content, the degree of abstraction related to mathematical concepts, principles and the degree of association between the mathematical concepts, as well as to the steps of reasoning and computing of the course content. The width of curriculum refers to the scope and width of the curriculum content, which is quantified by the number of contents. Time here refers to the time needed to complete the curriculum content, which can be quantified by lesson hours mentioned in the Mathematics Curriculum Standard. Shi's argument is that most students are able to understand the curriculum content as long as enough time is provided. From this point of view, they established a model for the measure of the degree of difficulty of mathematics curriculum: $N = \alpha \frac{S}{T} + (1 - \alpha) \frac{G}{T}$. In this model, N refers to the degree of difficulty of mathematics curriculum, S refers to the depth of curriculum, G refers to the width of curriculum, T refers to time, S / T is the comparable depth, G / T is the comparable width, a (0 <a <1) is the weighting coefficients reflecting the weighting degree of the comparable depth (S / T) or the comparable width G / T.

As mentioned above, there are currently quantitative researches on difficulty of mathematics courses or mathematical exercises, but there is neither quantitative study on difficulty of mathematics textbooks nor comparative study on difficulty of international mathematics textbooks. This study first constructed a model of the degree of difficulty of primary school mathematics textbooks, and examined the degree of difficulty of 12 sets of elementary school mathematics textbooks in 10 countries with the model constructed.

3.Reseach design

3.1 The selected primary school mathematics textbooks

In the present study, textbooks only refer to the books which are written based on curriculum standards, systematically reflecting content of the subject, excluding the teaching workbooks, teacher's manual, teaching reference books, educational software, and other teaching materials.

In this study, 12 sets of textbooks from ten countries in Asia, Europe, America, and Australia were chosen. Why these textbooks were chosen is that they are widely used and are valued as being influential in their own countries. Detailed information of the chosen textbooks is listed in Table 1 below.

Table 1 The selected primary school mathematics textbooks

Country	*Title*	*publishing house*	*Publishing year*	*Editor*	*Abbreviation*
China	Mathematics	People's Education Press	2009	Lu Jiang, Yang Gang	China (P)
Japan	わくわく算数	株式会社新興出版社啓林館	2010	清水静海, 船越俊介	Japan (Q)
South Korea	서울교육대학교	국정도서편찬위원회	2012	두산동아(주)	South Korea (D)
Singapore	My Pals are Here！Maths	Marshall Cavendish Education Publishers	2012	Fong Ho Kheong	Singapore (M)
U.K	Primary Maths	Harper Collins Education	2000	Peter Clarke	U.K.（H）
France	la clé des maths	Belin Publishers	2008	Gérard Champeyrache	France（B）
Russia	математики	21-го века ассоциация прессы (четвертый класс)	2010	Н.Б. Стоу Мина	Russia（T）
Germany	Welt Der Zahl	Schroedel Verlag GmbH	2010	Hans-Dieter Rinkens et al	Germany（Sc）
Germany	Das Zahlenbuch	Klett	2007	Gerhard N. Müller	Germany（Kl）
U.S	investigations	Person Education Companies	2008	Susan Jo Russell	U.S.（P）
Australia	New Signpost Maths for Victoria	Pearson Education Australia	2007	Alan McSeveny, Alan Parker	Australia（P）
Australia	Nelson Maths for Victoria	Thomson Nelson	2007	Tony Doyle	Australia（T）

3.2 The process of constructing the model

With the reference to the model suggested by Bao (2002) and Shi et al (2005) as reviewed above, a group of 12 experts at Southwest University worked together to identify possible factors which might influence the degree of difficulty of primary school mathematics textbooks. After this, project members further interviewed a group of highly experienced primary school mathematics teachers, primary school mathematics teaching researching officers, primary school mathematics textbook writers, and university mathematics education researchers. Basing on the project members' identification and interview results, 15 factors were finally determined which would influence the difficulty of primary school mathematics textbooks and a questionnaire was developed based on the 15 factors. 1236 primary mathematics teachers mainly from Chongqing, Sichuan, Hubei, Shandong were involved in the follow-up questionnaire survey. Factor analysis was further performed and five, four, and three main factors were identified. Finally, after a discussion by experts involved in the project, the three factors model was chosen to meet the purpose of the study:

$$N = f(C_1, C_2, E)$$

$$f = \alpha_1 C_1 + \alpha_2 C_2 + \alpha_3 E,\quad 0 < \alpha_1, \alpha_2, \alpha_3 < 1,\quad \alpha_1 + \alpha_2 + \alpha_3 = 1$$

$$C_2 = \alpha_{21} C_{21} + \alpha_{22} C_{22},\quad 0 < \alpha_{21}, \alpha_{22} < 1,\quad \alpha_{21} + \alpha_{22} = 1$$

$$E = \alpha_{31} E_1 + \alpha_{32} E_2,\quad 0 < \alpha_{31}, \alpha_{32} < 1,\quad \alpha_{31} + \alpha_{32} = 1$$

In this model, N refers to the degree of difficulty of primary mathematics textbooks, C_1 refers to the width of content (the amount of knowledge), C_2 refers to the depth of content, E refers to the degree of difficulty of exercises, C_{21} refers the style of presenting knowledge, C_{22} refers to the cognitive demand of knowledge, E_1 refers to the cognitive level of exercise, E_2 refers to the background of exercises, and $\alpha_1 = 0.2$, $\alpha_2 = 0.5$, $\alpha_3 = 0.3$; $\alpha_{21} = 0.5$、$\alpha_{22} = 0.5$; $\alpha_{31} = 0.6$, $\alpha_{32} = 0.4$.

In addition, after a discussion, this study formed some principles for the differentiation of knowledge. For the present of content, the cognitive demand of the content, and the cognitive demand of exercises, the study divided them into three levels and from easy to difficulty, 1,2,3 was assigned to each level respectively. Detailed information is listed in Table 2 below:

Table 2 principles for the differentiation of knowledge and exercise

the first stage dimension,	*the second stage dimension*	*level and score*		
		1	2	3
the depth of content	the presentating style of knowledge	intuiting	inducting	abstracting
	the cognitive demands of knowledge	knowing	understanding	applicating
the level of exercise	the cognitive demands of exercises	imitating	transfering	probing
	the background of exercises	no background	life background	Scientific background

Meanwhile, criterions were developed to reasonably determine the levels of difficulty by discussion. In order to make a reasonable comparison, C1and C2 were further standardized as followed:

1) C_1 is the ratio of the amount of knowledge in a country's textbook (D) and the amount of knowledge in the union of 10 countries' primary school mathematics textbooks (B), namely $C_1 = D / B$.

2) C_{21}, C_{22}, E_1, and E_2 are also standardized. For example, $C_{21} = \frac{n_1 \times 1 + n_2 \times 2 + n_3 \times 3}{n_1 + n_2 + n_3}$, n_1, n_2, n_3 refers to the amount of knowledge in the level of Intuiting, inducting, abstracting respectively. C_{22}、E_1、E_2 were standardized in a similar way.

3.3 Data collection

For the time constraint, this study only focused on the comparison of fourth grade textbooks in each of the chosen textbooks. In the process of analyzing textbook text, two to three researchers analyzed the same text with "the principle of dividing knowledge point ", "the evaluating criteria for the depth of content", "the evaluating criteria for the level of exercises" and the consistency between the researchers was further examined to make sure that each researcher can analyze texts according to the same standard as possible. The researchers then analyzed content in the text and its presentation, cognitive demanding basing on the demand of mathematics curriculum standards, analyzed the difficulty of each exercise and produced "the list of knowledge point", "the list of depth of every knowledge point", "the list of cognitive demanding levels of exercises ". Then, with the use of the model developed for the study, the difficulty of each dimension and the overall degree of the difficulty of each textbook were calculated.

4 Results and discussion

4.1 Values of the overall difficulty of textbooks

As shown in Table 3: the degrees of overall difficulty of textbooks used in France, Russia, Germany are generally lager than the ones in other countries. The degrees of overall difficult of primary school mathematics textbooks used in countries in East Asian, such as China, Japan and South Korea, are in the second stage, the degrees of the ones used in Singapore and Australia behind the intermediate of all the textbooks, while the degrees of overall difficult of textbooks used in the United Kingdom and the United States are smaller and are the lowest among the selected textbooks.

Table 3 Values of the overall difficulty of textbooks

No.	*textbooks*	*value*	*No.*	*textbooks*	*value*
1	France（B）	0.669	7	South Korea (D)	0.583

2	Russia（T）	0.605	8	Australia（T）	0.578
3	Germany（Sc）	0.594	9	Singapore (M)	0.564
4	Germany（Kl）	0.590	10	Australia（P）	0.556
5	China (P)	0.588	11	U.S.（P）	0.522
6	Japan (Q)	0.584	12	U.K.（H）	0.511

4.2 Values of the difficulty of each dimension

4.2.1 Values of the width of content

As shown in Table 4: the values of the width of content of primary school mathematics textbooks in Western countries (e.g., France, Germany, Australia) are generally larger than the other ones. The values of the width of content of primary school mathematics textbooks in East Asian countries (e.g., Japan, South Korean, Singapore, and China) are in or behind the intermediate of all the chosen textbooks, while the counterparts in United Kingdom and Russia are the smallest.

Table 4 Values of the width of content

No.	*textbooks*	*value*	*No.*	*textbooks*	*value*
1	France（B）	0.6091	7	Japan (Q)	0.4364
2	Germany（Sc）	0.555	7	South Korea (D)	0.4364
3	Australia（P）	0.5091	7	China (P)	0.4364
3	Australia（T）	0.5091	10	Singapore (M)	0.4
5	Germany（Kl）	0.491	11	Russia（T）	0.3727
6	U.S.（P）	0.445	12	U.K.（H）	0.318

4.2.2 Values of the depth of content

As shown in Table 5: the values of the depth of content in France is the biggest in the all textbooks, the counterparts in East Asian countries or European countries like South Korea, Japan, Russia, Germany, China, Singapore are in the intermediate stage, and the counterparts in Australia, the United Kingdom, and United States are generally smaller than the ones in other participant countries.

Table 5 Values of the depth of content

No.	*textbooks*	*value*	*No.*	*textbooks*	*value*
1	France（B）	0.7414	7	Singapore (M)	0.6705
2	South Korea (D)	0.712	8	Germany（Sc）	0.67
3	Japan (Q)	0.7037	9	Australia（P）	0.628
4	Russia（T）	0.7033	10	U.K.（H）	0.61
5	Germany（Kl）	0.686	11	Australia（T）	0.6042
6	China (P)	0.684	12	U.S.（P）	0.517

4.2.3 Values of the difficulty of exercises

As shown in Table 6: the values of difficulty of exercises in the countries like Russian, France, and the United States are closed to each other, which are generally bigger than the ones in other participant countries. The values of difficulty of exercises in China is in the second stage in the all textbooks. The values of difficulty of exercises in the United Kingdom and South Korean are comparatively smaller than the ones in other participant countries. Furthermore, the values of difficulty of exercises of different versions in Australian are quite different.

Table 6 Values of the difficulty of exercises

No.	*textbooks*	*value*	*No.*	*textbooks*	*value*
1	Russia（T）	0.5942	6	Singapore (M)	0.4971
2	France（B）	0.5871	8	Germany（Sc）	0.493
3	U.S.（P）	0.581	9	Japan (Q)	0.4819
4	Australia（T）	0.5797	10	U.K.（H）	0.474
5	China (P)	0.53	11	Australia（P）	0.4684
6	Germany（Kl）	0.497	12	South Korea (D)	0.465

5 Conclusion

Through the comparison of the degree of difficulty of the 12 selected primary school mathematics textbooks used in the ten countries, the following conclusion could be made based on the findings reported above:

1) The degrees of overall difficulty of the 12 sets of elementary school mathematics textbooks are in a certain geographical proximity in its distribution.

For example, textbooks in the first level of overall difficulty are in France, Russia, Germany in continental Europe, textbooks in the second level of overall difficulty are in China, Japan and South Korea in East Asia, textbooks in the third level of overall difficulty are in Singapore and Australia, and the overall difficulty of textbooks in United States and United Kingdom ranks last. People cannot help but suspect that the degrees of overall difficulty of mathematics textbooks for elementary schools have greater relevance with the geographical culture.

2)In the study, the values of the width of content of primary school mathematics textbooks in China, Japan, South Korea, and Russia are in or behind the intermediate of all the chosen textbooks, while the values of the depth of primary school mathematics textbooks in those countries rank at or near the top .In contrast, along with lower teaching requirements, the values of the width of content of primary school mathematics textbooks in United States and Australia are larger than the ones in China, Japan, South Korea, and Russia. This suggests that generally, primary school mathematics textbooks used in China, Japan, South Korea, and Russia cover less contents but are more difficult than their counterparts in United States or Australia. It is possibly because of the different orientations on the value of mathematics education in the two sorts of countries above: China et al emphasis on learning more deeply, Australia et al emphasis on learning more widely.

3) In terms of the degree of overall difficulty, the width of the contents, or the difficulty of exercises, the degrees of the primary mathematics textbooks in Asian countries like China, Japan, South Korea and Singapore are in or behind the middle places among the participant textbooks. So, the fact that pupils in East Asia achieved better in TIMSS or PISA than their counterparts in other countries is not mainly caused by the reason that the primary school mathematics textbooks or mathematical exercises in textbooks in East Asia are too difficult but by other reasons .In fact, the degree of overall difficulty of elementary school mathematics textbooks in France, Germany, Russia are among the top three, but pupils in those countries achieved lower scores in TIMSS or PISA than the counterparts in East Asian countries.

Similarly, it also implies that the mathematics learning burden of East Asian primary school students is not mainly caused by the reason that East Asian primary school mathematics textbooks are too difficult. With the perspective of region\culture, it further illustrates that as identified in some international comparative studies on mathematics education, China, Japan, South Korea and Singapore are CHC countries, hardship, exam-orientation, parents' high expectation et al are existed in the culture, leading to the increasing difficulty by teachers in practice, which are quite different from Western countries.

Reference

1.Bao Jiansheng (2002). A comparison on the comprehensive difficulty of Sino-British middle school mathematics curriculum expectations. *Global Education*, 31,48-52.

2.Chall,J.S.,Conard,S.S. & Harris-Sharples S.(1991). *Should textbooks challenge students? the case of easier or harder books.* NY: Teachers College Press, Columbia University.

3.Conard Sue Stankewitz(1981). The Difficulty of Textbooks for the Elementary Grades: A Survey of Educators' and Publishers' Preferences. Research prepared at Harvard Graduate School of Education.

4.Davison Alice(1986). Readability and Questions of Textbook Difficulty. *Reading Education Report*. No. 66.

5.Huang Puquan (1995) . Exploration for a gray model GM (1, 1) on the difficulty degree for primary and secondary school course, *Systems Engineering Theory and Practice*, 84,63-70.

6.Maricela Corzo-Pena(1996).textbook difficulty and syntactic complexity:an analysis of grades three and four scicence textbooks. Unpublished MA dissertation, University of Havana.

7.Nohara,D.(2001).Acomparison of the National Assessment of Educational Progress(NAEP),.theThird Inrernarional Mathematics and Science Study Repeat(TIMSS-R),and rhe Programme for International Student Assessment(PISA).NECS Working Paper.No.2001-07.

8.Shi Ningzhong, Kong Fanzhe & Li Shuwen (2005). Course Difficulty model: China Compulsory Geometry Course difficulty comparison. *Northeast Normal University (Philosophy and Social Sciences)*, 218,51-55.