Regression with Autocorrelated Errors Using Design-Adapted Haar Wavelets

Rogério F. Porto
Bank of Brazil, Brasília, Brazil rogerio@bb.com.br

Pedro A. Morettin
University of São Paulo, São Paulo, Brazil pam@ime.usp.br

Elisete C. Q. Aubin*
University of São Paulo, São Paulo, Brazil aubin@ime.usp.br

There are several approaches to non-parametric regression using wavelet methods. In most of them, the errors are independent and identically distributed. (IID) Gaussian variables and the design, though unequally spaced, is fixed instead of random. Essentially, they adapt methods and wavelets created for equally spaced samples to the unequally spaced design. However, some recent methods employ a basis formed by the so called unbalanced Haar wavelets. They have been used to solve the problem of unequally spaced samples, mainly because the resulting basis can be selected to be adapted to the design. However, the research with design-adapted Haar wavelets has considered almost only the case of IID errors and formal results taking into account the presence of stationary dependent errors are considered an open issue. In this presentation we investigate wavelet shrinkage for very general unequally sampled designs in the presence of correlated errors. For a class of continuous functions, we give sufficient conditions in order to retain the good properties of wavelet estimators but in the presence of stationary dependent errors. In fact, we investigate the behavior of a linear estimator as well as a nonlinear estimator. Specifically we show that for absolutely summable error covariances, the risk of the linear estimator decays slower when the errors are correlated than when they are independent. For nonlinear estimators, we are able to show that, if additionally the errors are strong mixing with a specific coefficient, then its risk decays as fast as when the errors are IID. Thus, the nonlinear estimator is not as general as the linear one, but its risk decays faster. Some simulation results are presented in order to highlight the behavior of the estimators for finite samples.

Key Words: Denoising, semi-parametric estimation, smoothing, unbalanced Haar wavelets