

**Estimation of the marginal expected shortfall:
the mean when a related variable is extreme**

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Denote the loss return on the equity of a financial institution as X and that of the entire market as Y . For a given very small value of $p > 0$, the marginal expected shortfall (MES) is defined as $E(X | Y > Q_Y(1 - p))$, where $Q_Y(1 - p)$ is the $(1 - p)$ -th quantile of the distribution of Y . The MES is an important factor when measuring the systemic risk of financial institutions. For a wide nonparametric class of bivariate distributions, we construct an estimator of the MES and establish the asymptotic normality of the estimator when $p \downarrow 0$, as the sample size $n \rightarrow \infty$. Since we are in particular interested in the case $p = O(1/n)$, we use extreme value techniques for deriving the estimator and its asymptotic behavior. The finite sample performance of the estimator and the adequacy of the limit theorem are shown in a detailed simulation study. We also apply our method to estimate the MES of three large U.S. investment banks.

Key Words : Asymptotic normality, extreme values, tail dependence.